GNSS basic2

Tokyo University of Marine Science and Technology
Nobuaki Kubo

主な引用先

- 精説GPS（測位航法学会）
- よくわかる衛星測位と位置情報（日刊工業新聞社）
- 内閣府準天頂衛星のHP
- 高須様のHP
- Dr. Feng-Yu Chu氏の資料

- 上記以外は個別に記載

Contents

- Introduction
- Coordinate Systems
- Satellite Position
- Measurements and Errors

Basic 1

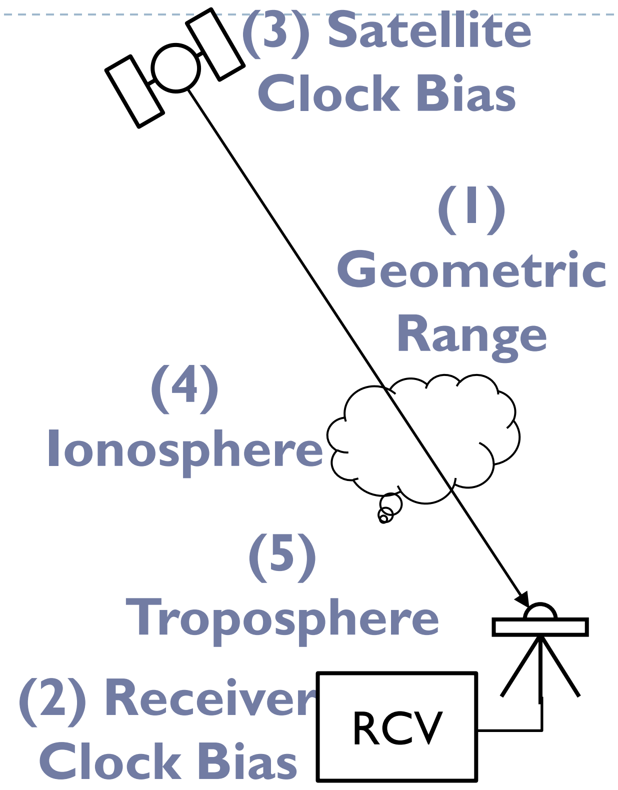
- Calculating Position
- Improved Position (DGNSS, RTK, PPP)
- QZSS

Basic2

79問の確認用のGoogleフォームでの4択問題があります
70問以上正解してください

Pseudo-range Model

$$
\begin{aligned}
P_r^s &\equiv c\tau \\
&= c(\bar{t}_r - \bar{t}^s) \\
&= c((t_r + dt) - (t^s + dT^s)) + \varepsilon_P \\
&= c(t_r - t^s) + c(dt_r - dT^s) + \varepsilon_P \\
&= (\rho_r^s + I_r^s + T_r^s) + c(dt_r - dT^s) + \varepsilon_P \\
&= \underbrace{\rho_r^s}_{(1)} + c(\underbrace{dt_r}_{(2)} - \underbrace{dT^s}_{(3)}) + \underbrace{I_r^s}_{(4)} + \underbrace{T_r^s}_{(5)} + \underbrace{\varepsilon_P}_{(6)}
\end{aligned}
$$



(3) Satellite Clock Bias

(1) Geometric Range

(4) Ionosphere

(5) Troposphere

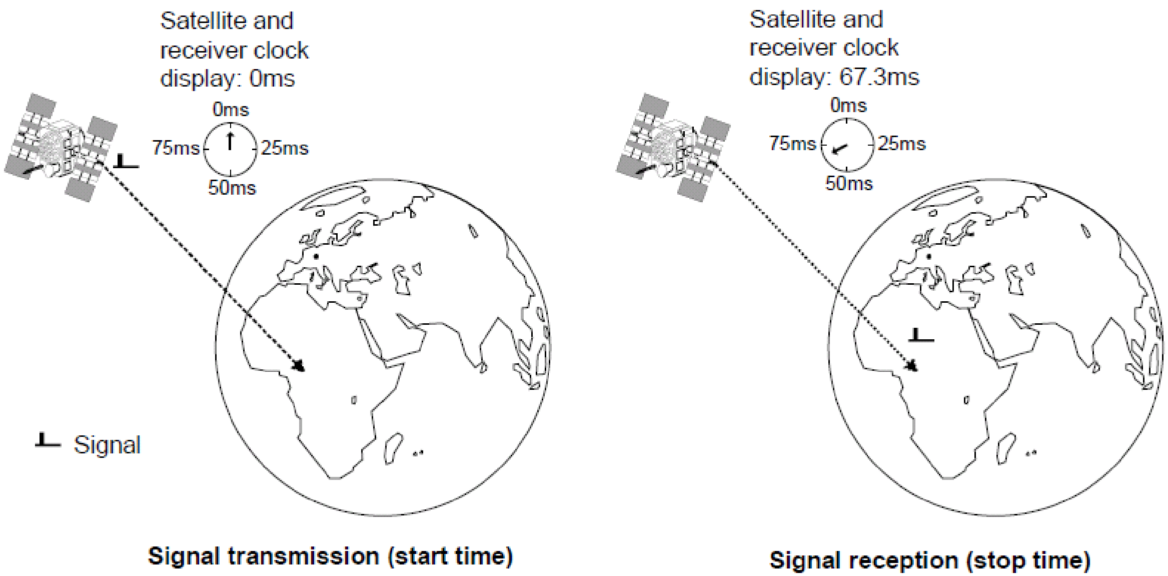
(2) Receiver Clock Bias

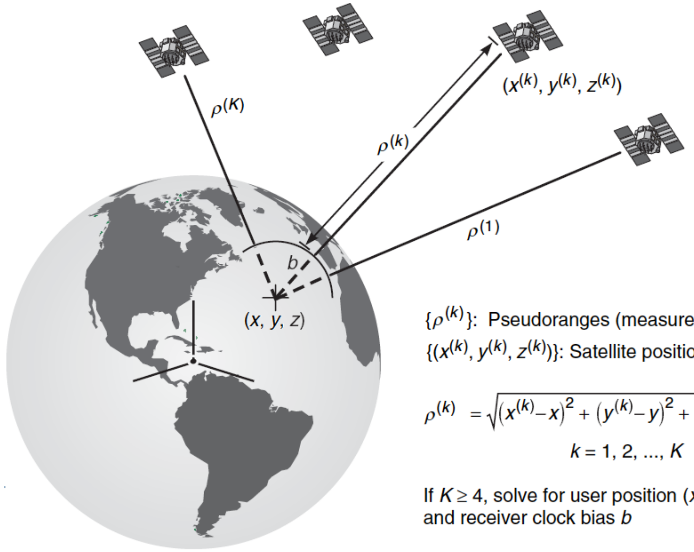
RCV

(6) Multipath + Receiver Noise + Other Errors

The reason why we call "pseudo-range" is from (2).
How many unknown parameter do we have ?

x, y, z, receiver clock offset





$\{\rho^{(k)}\}$: Pseudoranges (measurements)

$\{(x^{(k)}, y^{(k)}, z^{(k)})\}$: Satellite positions (known)

$$\rho^{(k)} = \sqrt{(x^{(k)}-x)^2 + (y^{(k)}-y)^2 + (z^{(k)}-z)^2} - b$$

$$k = 1, 2, \ldots, K$$

If $K \geq 4$, solve for user position (x, y, z), and receiver clock bias b

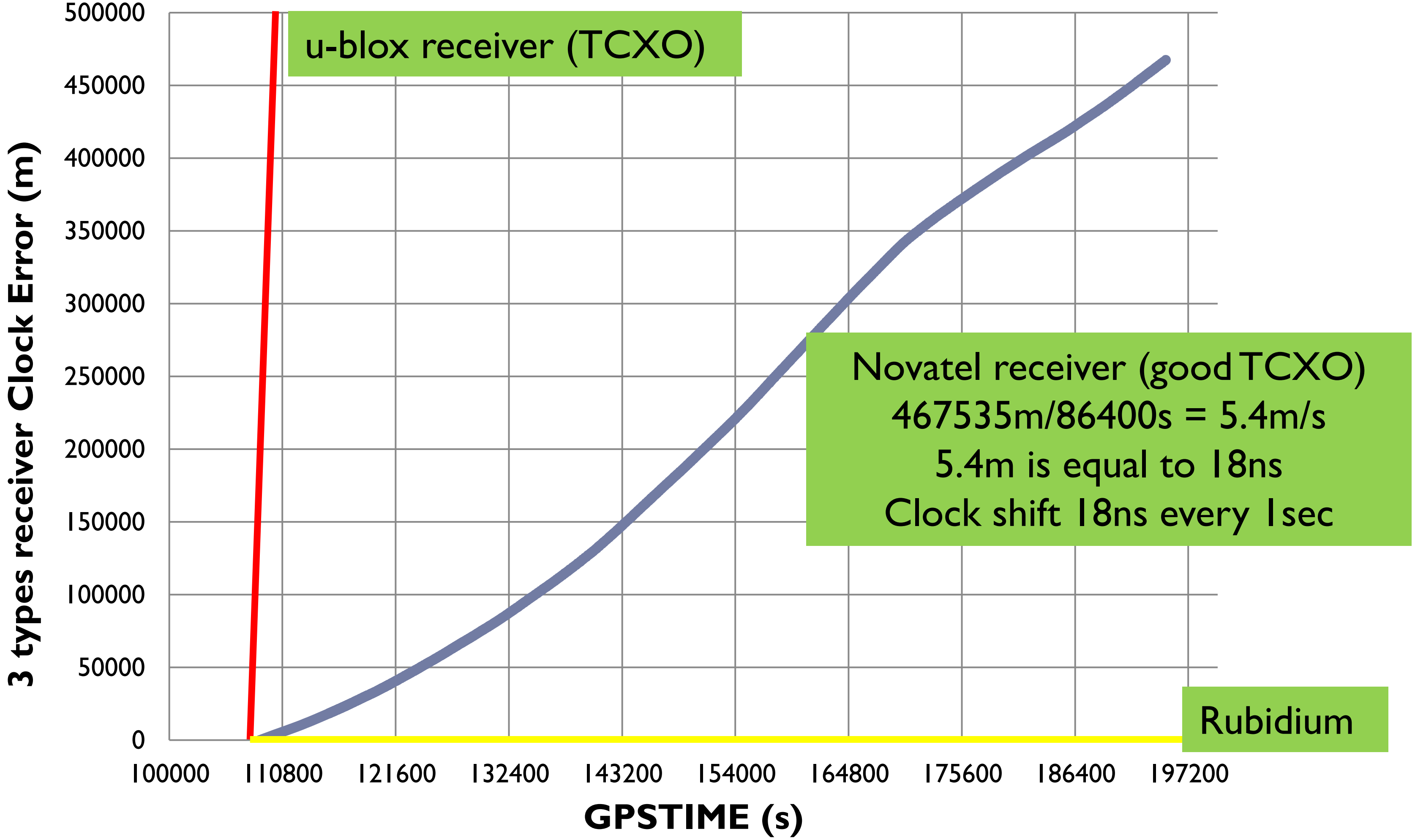
- Satellite clock is corrected using navigation data.
- Fortunately, receiver clock offset is same for all satellites.
- Therefore, unknown variables should be solved are x, y, z and receiver clock offset.

クロック推定精度は位置の精度とほぼ同等→3mで10ns程度

三辺測量

What is receiver clock offset ?

Receiver clock offset is coproduct of single positioning

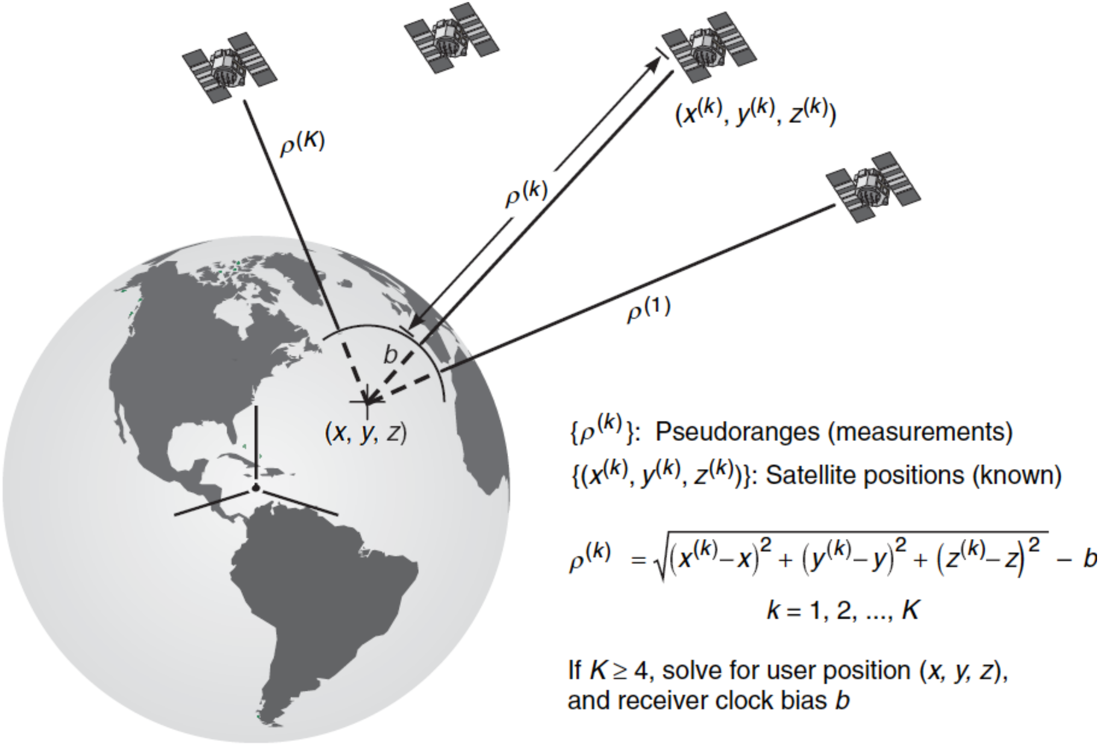


Novatel receiver (good TCXO)
467535m/86400s = 5.4m/s
5.4m is equal to 18ns
Clock shift 18ns every 1sec

原子時計の安定度は$10^{-12\text{から}-13}$

Single Point Positioning

- Using the code measurements only to compute the absolute position of the receiver in the ECEF system.
- Satellite clock can be corrected using navigation data.
- Fortunately, receiver clock offset is same for all satellites.
- Other measurement errors are assumed negligible, being absorbed in parameter of the receiver clock error.
- Therefore, there are 4 unknown parameters here. They are three range components x, y, z and the receiver clock error.
- LSE is used to computed the unknown parameters.



$\{\rho^{(k)}\}$: Pseudoranges (measurements)

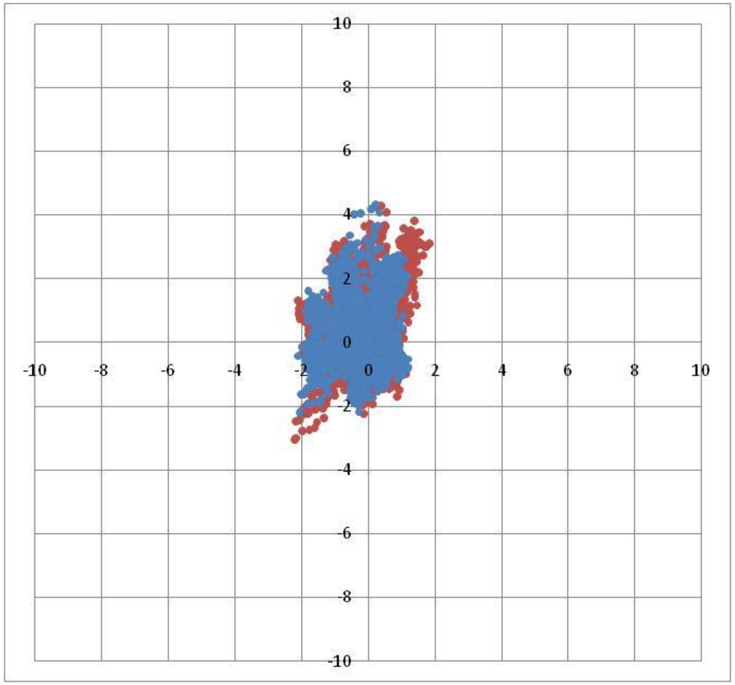
$\{(x^{(k)}, y^{(k)}, z^{(k)})\}$: Satellite positions (known)

$$\rho^{(k)} = \sqrt{(x^{(k)}-x)^2 + (y^{(k)}-y)^2 + (z^{(k)}-z)^2} - b$$

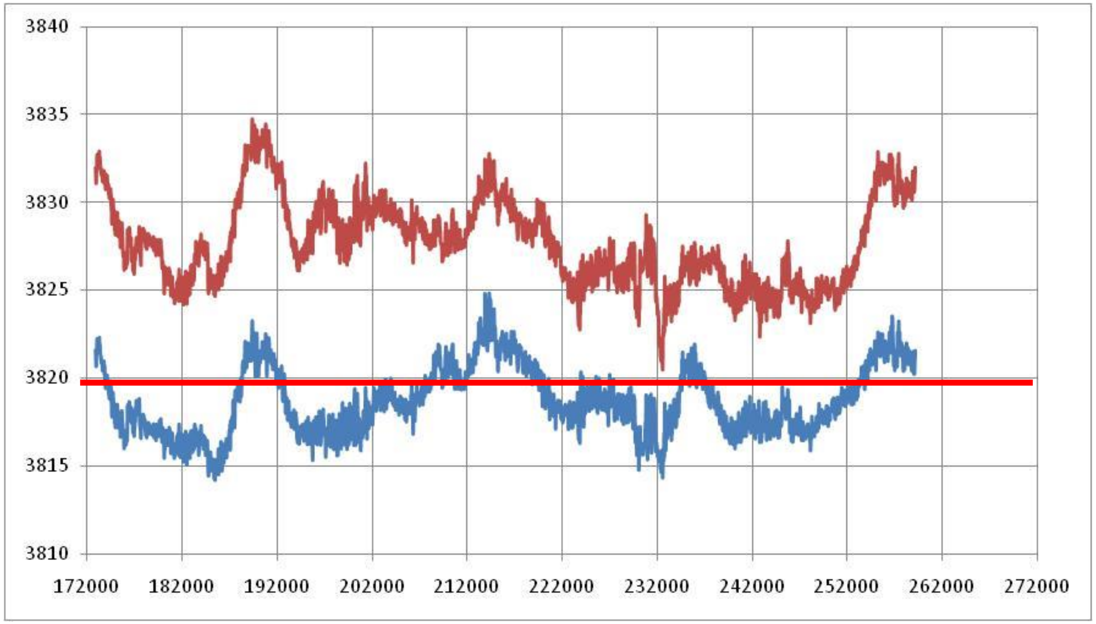
$$k = 1, 2, \ldots, K$$

If $K \geq 4$, solve for user position (x, y, z), and receiver clock bias b

Single point positioning at Mt. Fuji (6/1/2010)



Horizontal Errors (m)



Height Errors (m)

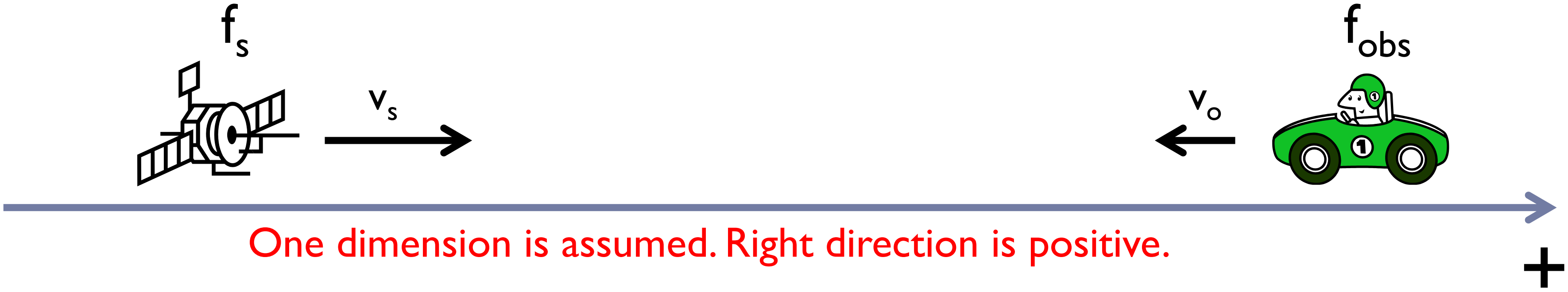
Blue : Stand Alone Positioning

Red : Stand Alone Positioning without Iono and Tropo Estimation

Demonstration of SPP

- Excel is used.

Doppler Effect



One dimension is assumed. Right direction is positive.

- Receiver is set in the car.
- Received frequency is
- “cs” is speed of light.

$$f_{obs} = f_s \frac{cs - v_o}{cs - v_s}$$

- Doppler frequency “f_D” is equal to “$f_{obs} - f_{source}$”
- FLL (frequency lock loop) tries to estimate “f_D”.
- Once we can estimate “f_D”, “v_o” can be resolved.

Velocity Estimation

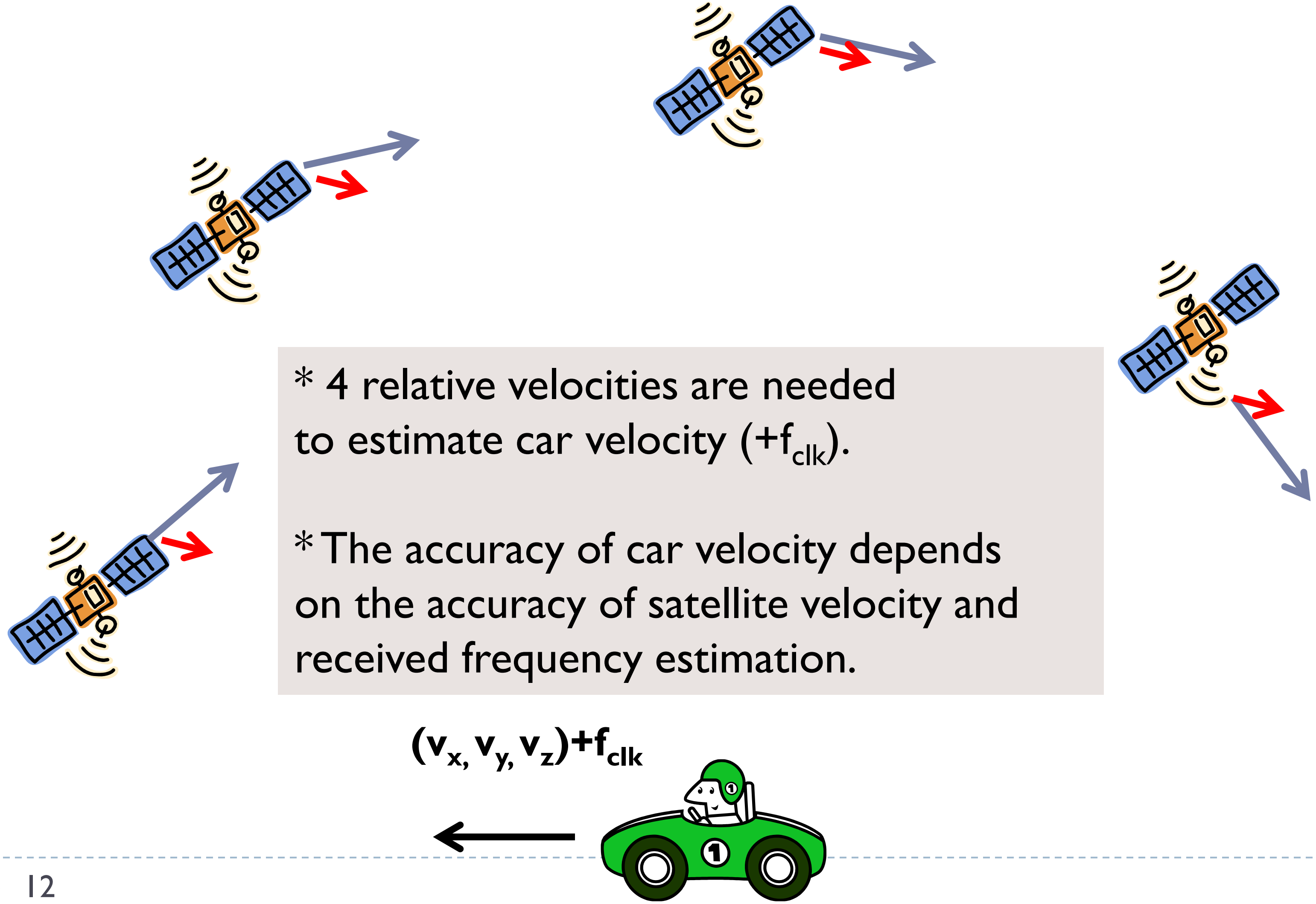
- Velocity estimation in GPS is just same as shown in the previous slide.
- The differences are as follows.

* **3 dimension velocity ($v_{x,}$ $v_{y,}$ v_z) have to be estimated.**

* **Frequency in the receiver is based on on-board clock.**

- 4 unknown variables ($v_{x,}$ $v_{y,}$ $v_{z,}$$f_{clk}$) have to be estimated using at least 4 visible satellites. DOP is also important.
- Velocity estimation is same as position estimation.

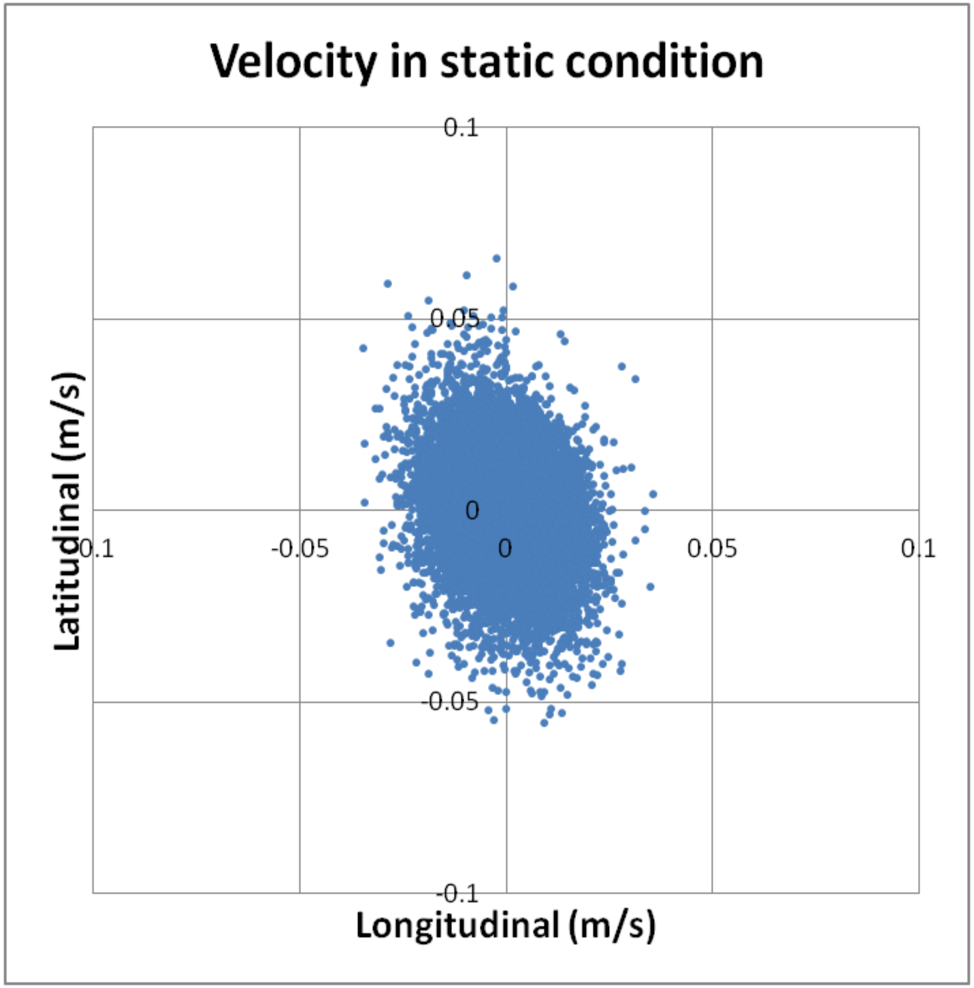
Image of Velocity Estimation



* 4 relative velocities are needed to estimate car velocity ($+f_{clk}$).

* The accuracy of car velocity depends on the accuracy of satellite velocity and received frequency estimation.

Performance of GPS based Velocity



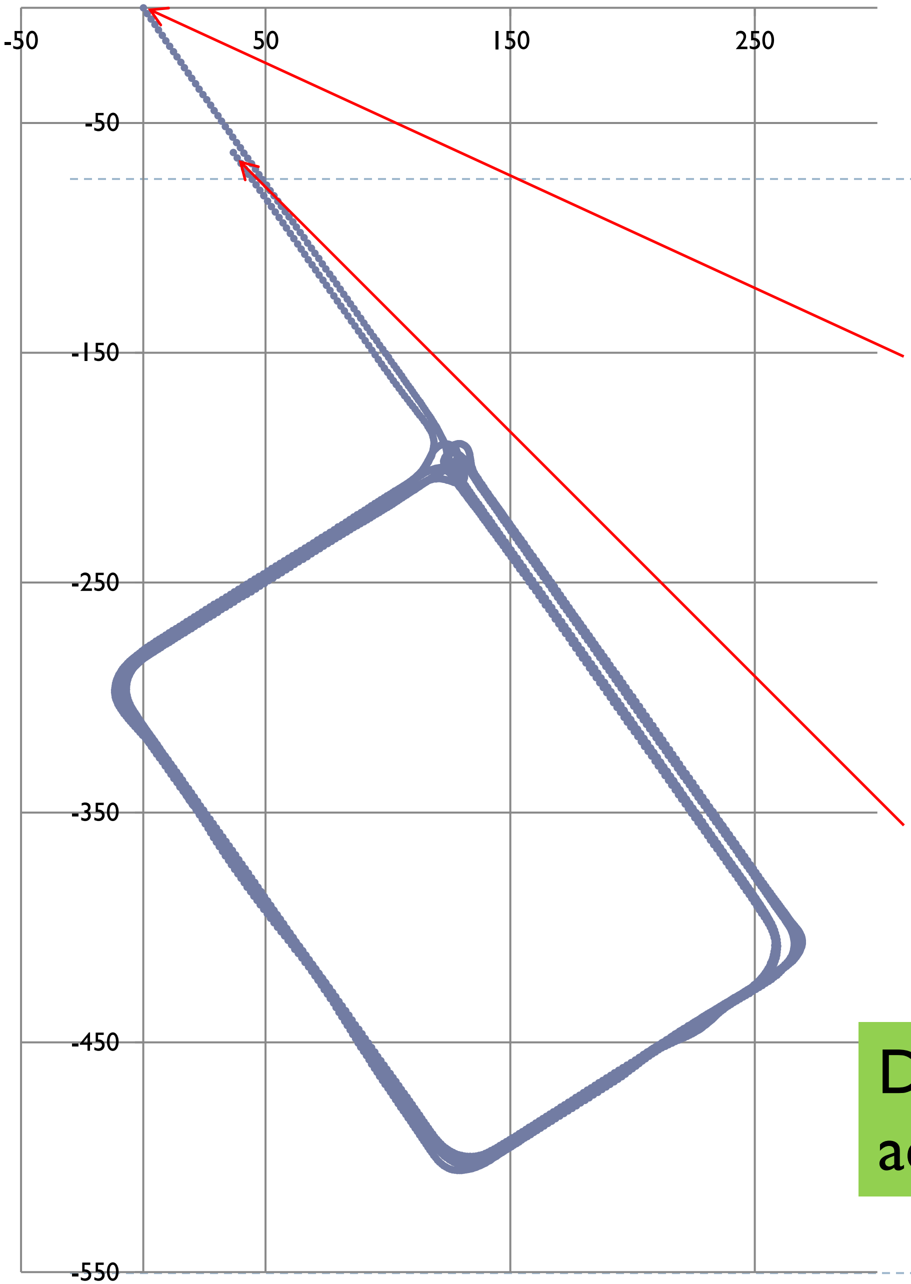
std = 1.6 cm/s

Accuracy in terms of frequency
GPS L1 wavelength = 19cm
1Hz : 19cm
0.1Hz : 1.9cm

Accuracy in terms of satellite velocity
sv_vel [t]=(sv_vel [t+1]-sv_vel [t-1])/2
based on ephemeris parameters
Accuracy is quite good.

Moving Platform

(Koto-ku Ariake)

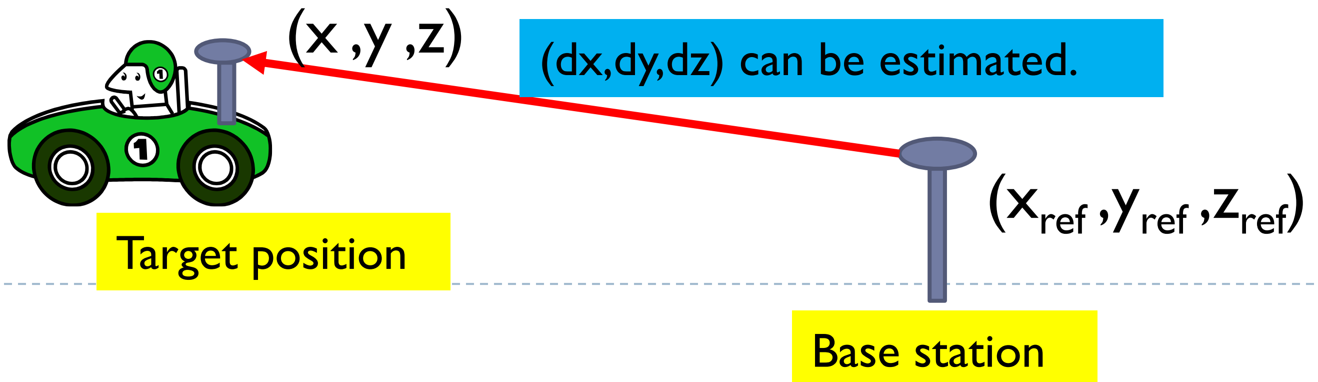


- Origination : 0,0
- Velocity was accumulated.
- Data Rate : 5Hz
- Period : 650 sec
- Receiver : NovAtel OEM6
- Left and right rounds : 6 times
- End point : 36.76m,-62.91m
- RTK : 35.75m,-65.18m

Deviation after 11 minutes velocity accumulation was about 2-3 m.

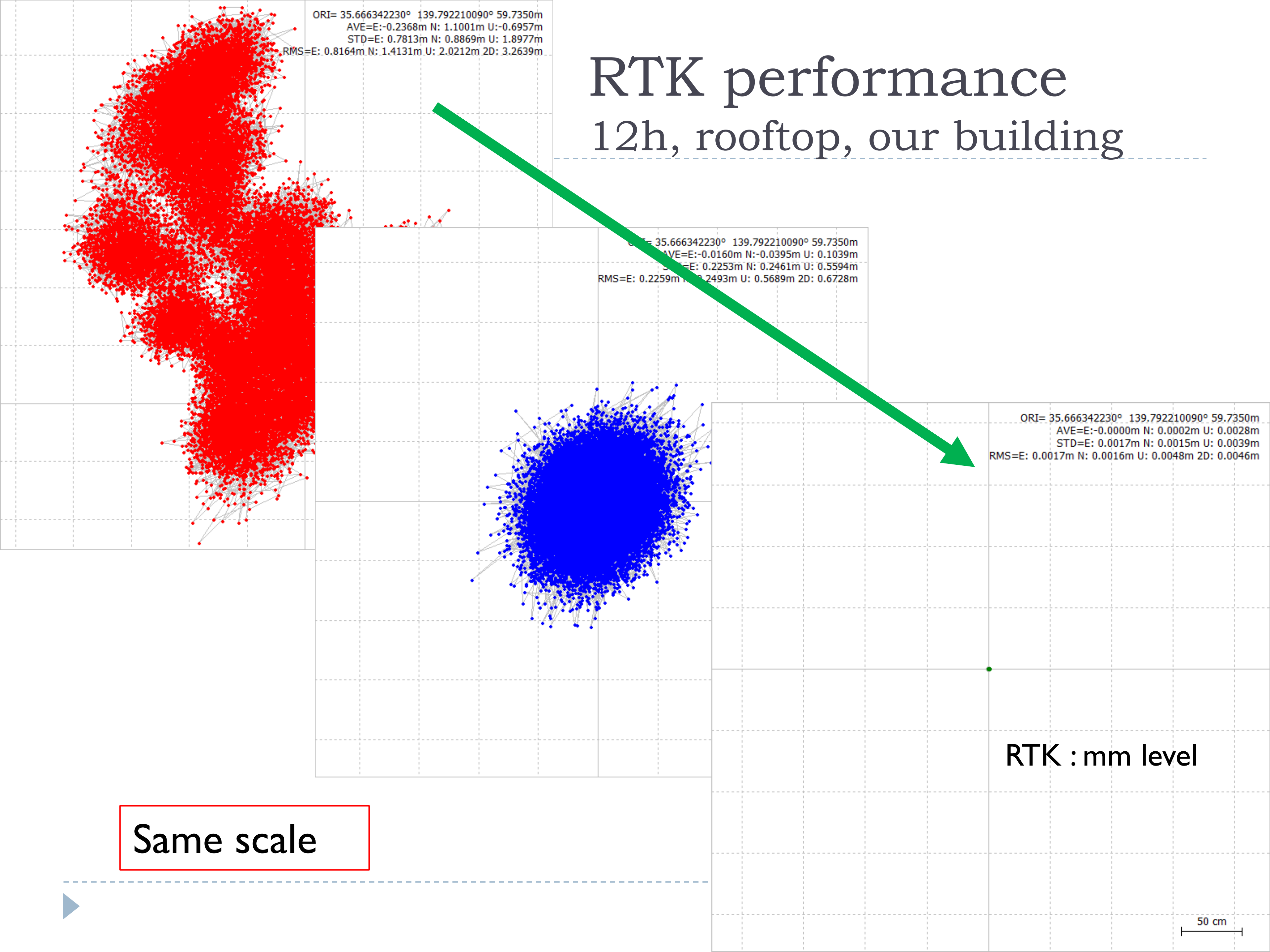
Improved GPS (relative positioning)

- DGPS and RTK are powerful method for error mitigation.
- DGPS uses the fact that the most of error sources change slowly in the time domain if the distance between reference and user is approx. within 100km.
- Please remember **that differential technique provides only vector solution** from base station to the target position→precise position of base station should be prepared.



RTK performance

12h, rooftop, our building



```
ORI= 35.666342230°  139.792210090° 59.7350m
AVE=E:-0.2368m N: 1.1001m U:-0.6957m
STD=E: 0.7813m N: 0.8869m U: 1.8977m
RMS=E: 0.8164m N: 1.4131m U: 2.0212m 2D: 3.2639m
```

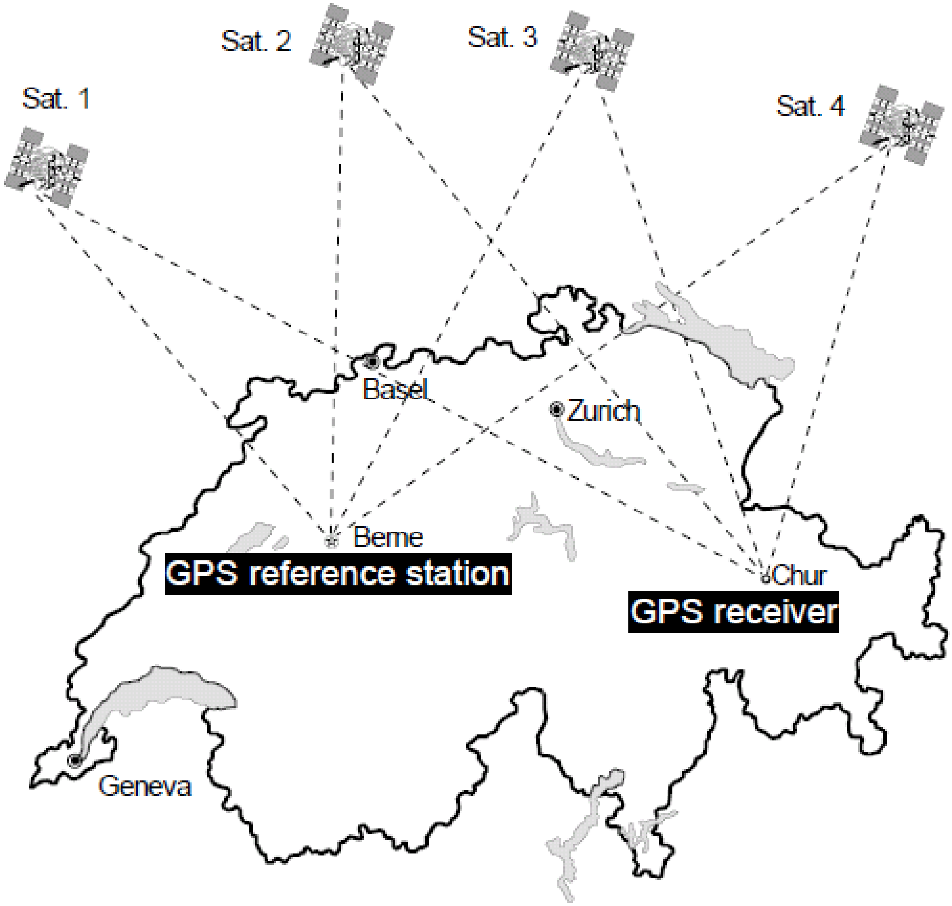
```
= 35.666342230°  139.792210090° 59.7350m
VE=E:-0.0160m N:-0.0395m U: 0.1039m
=E: 0.2253m N: 0.2461m U: 0.5594m
RMS=E: 0.2259m     2493m U: 0.5689m 2D: 0.6728m
```

```
ORI= 35.666342230°  139.792210090° 59.7350m
AVE=E:-0.0000m N: 0.0002m U: 0.0028m
STD=E: 0.0017m N: 0.0015m U: 0.0039m
RMS=E: 0.0017m N: 0.0016m U: 0.0048m 2D: 0.0046m
```

RTK : mm level

Same scale

Image of DGPS



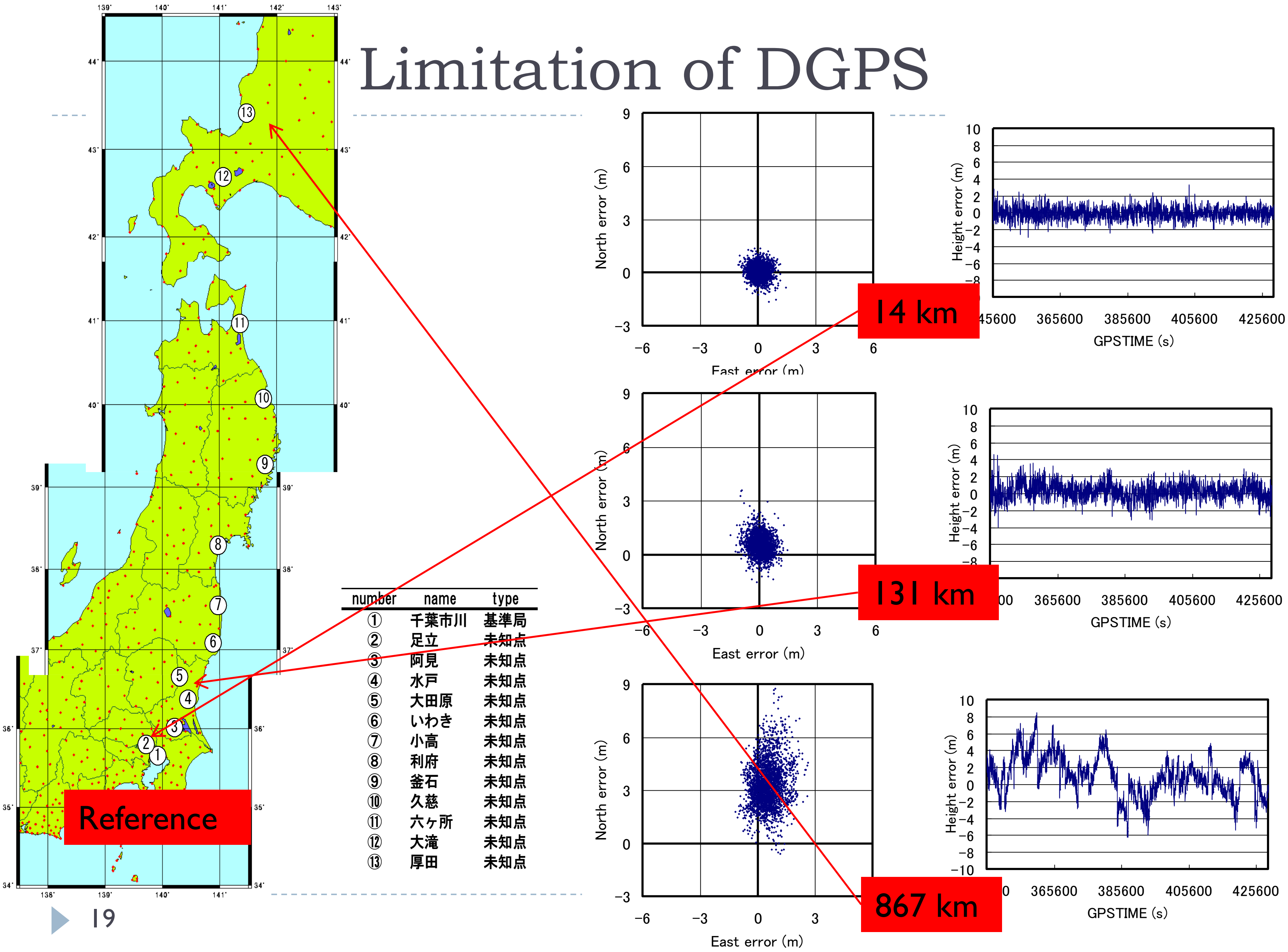
- Determination of the correction values at the reference station
- Transmission of the correction values from the reference station to the GPS user
- Compensation for the determined pseudo-ranges to correct the calculated position of the GPS user

Correction [prn] = Pseudo-range[prn] – True-range [prn]

DGPS mitigates …

Source	Potential error size	Error mitigation using DGPS
Satellite clock model	2 m (rms)	0.0 m
Satellite ephemeris prediction	2 m (rms) along the LOS	0.1 m (rms)
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	0.2 m (rms)
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	0.2 m (rms) + altitude effect
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	→
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	→

Limitation of DGPS



number	name	type
①	千葉市川	基準局
②	足立	未知点
③	阿見	未知点
④	水戸	未知点
⑤	大田原	未知点
⑥	いわき	未知点
⑦	小高	未知点
⑧	利府	未知点
⑨	釜石	未知点
⑩	久慈	未知点
⑪	六ヶ所	未知点
⑫	大滝	未知点
⑬	厚田	未知点

Reason for Accuracy Deterioration

- The main reason for the accuracy deterioration is atmospheric errors.
- For example, the tropospheric errors between Tokyo and Sapporo will be different (due to the difference of satellite elevation). The amount of error depends on the elevation angle.
- Normal DGPS doesn't take into the above error account at all.

RTK (Real Time Kinematic)

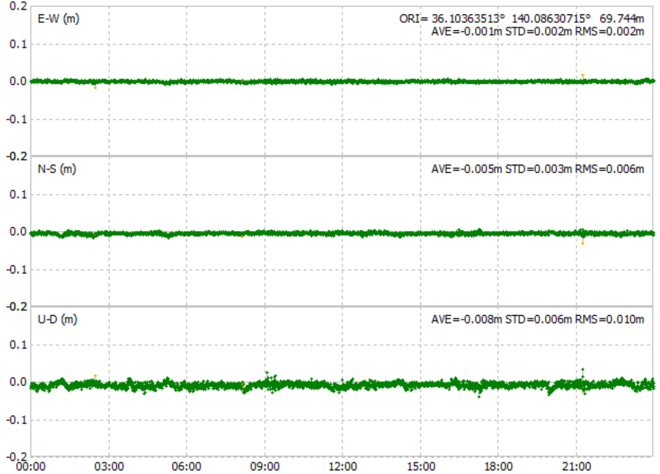
- The concept of RTK is same as DGPS.
- RTK uses carrier phase measurements. DGPS uses pseudo-range measurements.
- GPS receiver is able to measure 1/100 of wavelength of L1 frequency (19 cm).
- If you have high-end receiver, you know your position within 1-2cm accuracy as long as you have 5 or more LOS satellites.

Real-time Kinematic

- **Technique with Carrier-based Relative Positioning**
  - Real-time Position of Rover Antenna
  - Transmit Reference Station Data to Rover via Comm. Link
  - **OTF** (On-the-Fly) Integer Ambiguity Resolution
  - Typical Accuracy: 1 cm + 1ppm x BL RMS (Horizontal)
  - Applications:
    Land Survey, Construction Machine Control, Precision Agriculture etc.

Effect of Baseline Length

BL=0.3 km

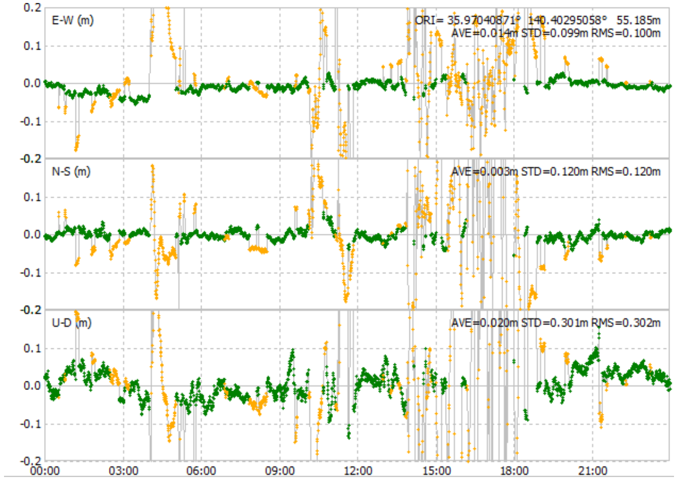


RMS Error:
E: 0.2cm
N: 0.6cm
U: 1.0cm
Fix Ratio: 99.9%

BL=13.3 km

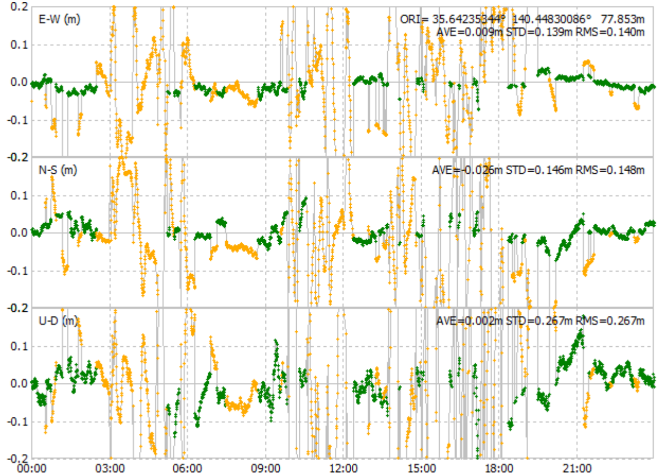
RMS Error:
E: 2.2cm
N: 2.4cm
U: 10.6cm
Fix Ratio: 94.2%

BL=32.2 km



RMS Error:
E: 10.0cm
N: 12.0cm
U: 30.2cm
Fix Ratio: 64.3%

BL=60.9 km



RMS Error:
E: 14.0cm
N: 14.8cm
U: 26.7cm
Fix Ratio: 44.4%

(24 hr Kinematic ●: Fixed Solution ●: Float Solution)

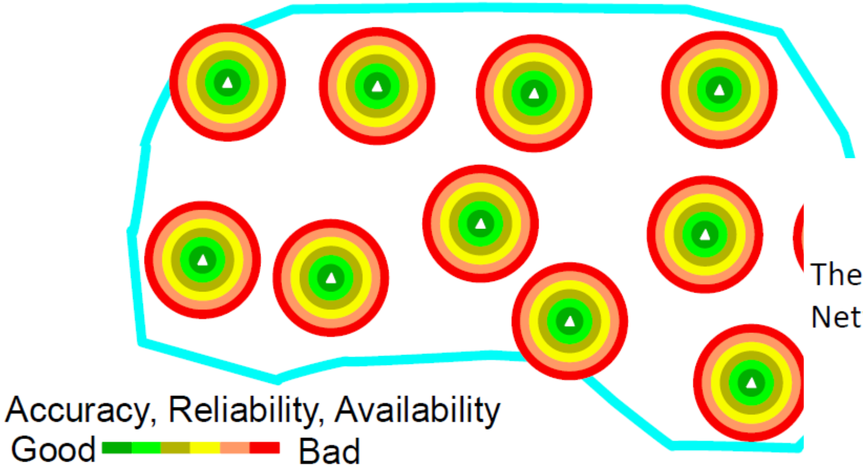
Network RTK (NRTK)

- **Extension of RTK**
  - RTK without User Reference Station
  - Sparse Networked Reference Stations
  - Correction Messages via Mobile-Phone Network
  - Format: **VRS**, **FKP**, MAC, RTCM 2.3, RTCM 3.1
  - Server S/W: Trimble GPSNet, GEO++ GNSMART, ...
  - NTRIP Networked Transport of RTCM via Internet Protocol
- **NRTK Service in Japan**
  - GEONET: ~1200 Reference Stations by GSI
  - NGDS, JENOBA, Terasat

Concept of NRTK

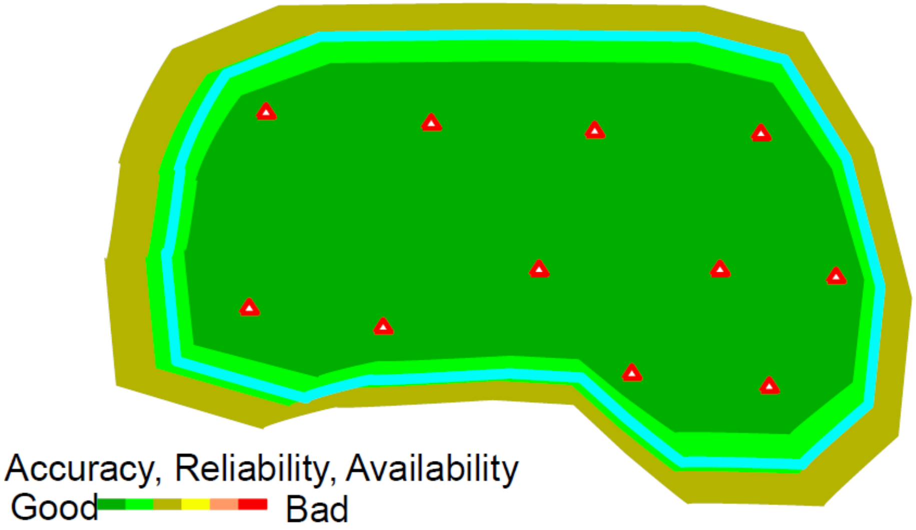
Network of Individual Reference Stations

To cover a large area with single reference stations to run RTK, we need multitude of points and still we have huge gaps between the points.



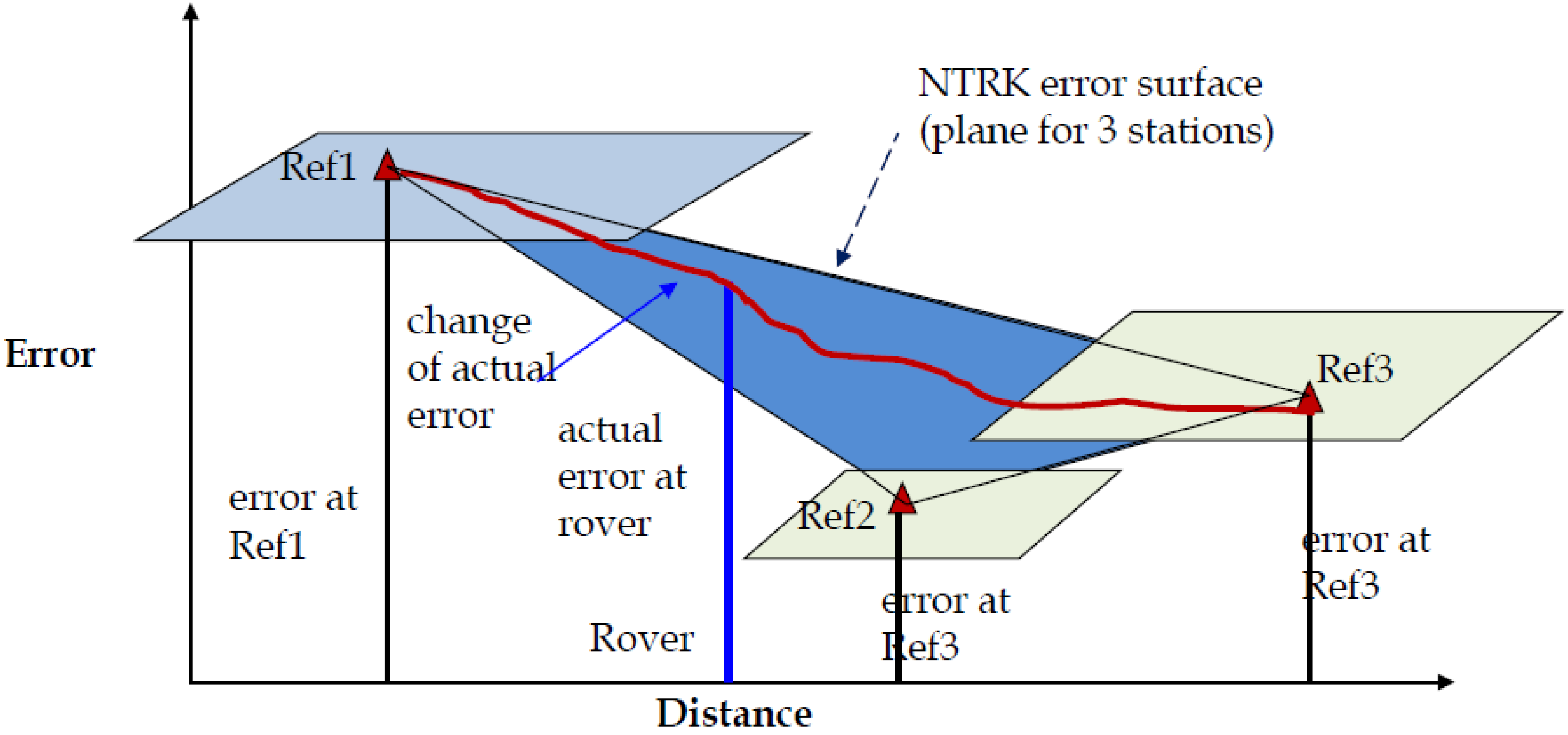
The Solution is Network RTK (NRTK)!

The same area is covered with much less number of points using the Network RTK concept. All the area is covered with no gaps.



Source: GEO++

Relationship between Errors



Several interpolation algorithms

Source: GEO++

Actual Steps of RTK

- After this summer school, please check the followings regarding the process of RTK to deepen your understanding !

1. Generating "double difference"
2. Finding "integer ambiguities"
3. Baseline processing

1. DD (Double Difference)

$$\Phi_{ub}^{ij} \equiv \lambda((\phi_u^i - \phi_b^i) - (\phi_u^j - \phi_b^j))$$
$$= \rho_{ub}^{ij} + c(dt_{ub}^{ij} - dT_{ub}^{ij}) - I_{ub}^{ij} + T_{ub}^{ij} + \lambda N_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_{\Phi}$$
$$= \rho_{ub}^{ij} - I_{ub}^{ij} + T_{ub}^{ij} + \lambda N_{ub}^{ij} + d_{ub}^{ij} + \varepsilon_{\Phi}$$

$$dt_{ub}^{ij} = dt_u^{ij} - dt_b^{ij} = 0, \boxed{dT_{ub}^{ij} = dT_{ub}^{i} - dT_{ub}^{j} \approx 0}$$

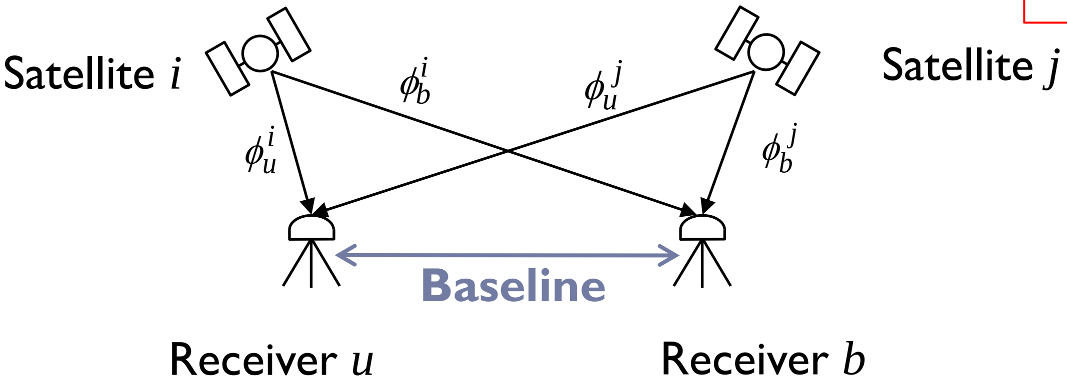
$$B_{ub}^{ij} = (\phi_{u,0} - \phi_0^i + N_u^i) - (\phi_{b,0} - \phi_0^i + N_b^i) - (\phi_{u,0} - \phi_0^j + N_u^j) + (\phi_{b,0} - \phi_0^j + N_b^j) = N_{ub}^{ij}$$

Without reference station,
it is impossible to remove “receiver
And satellite clock error” completely !
Generate new observation
which means double difference.

(short Baseline and same antenna type)

$$I_{ub}^{ij} = I_{ub}^{i} - I_{ub}^{j} \approx 0, T_{ub}^{ij} = T_{ub}^{i} - T_{ub}^{j} \approx 0, d_{ub}^{ij} = d_{ub}^{i} - d_{ub}^{j} \approx 0$$

Why do we say the
baseline limitation of RTK ?
(10-100 km or more)
It strongly depends on
each RTK engine !



Memo for Misra & Enge:
http://gpspp.sakura.ne.jp/diary200608.htm

2. Integer Ambiguity Resolution

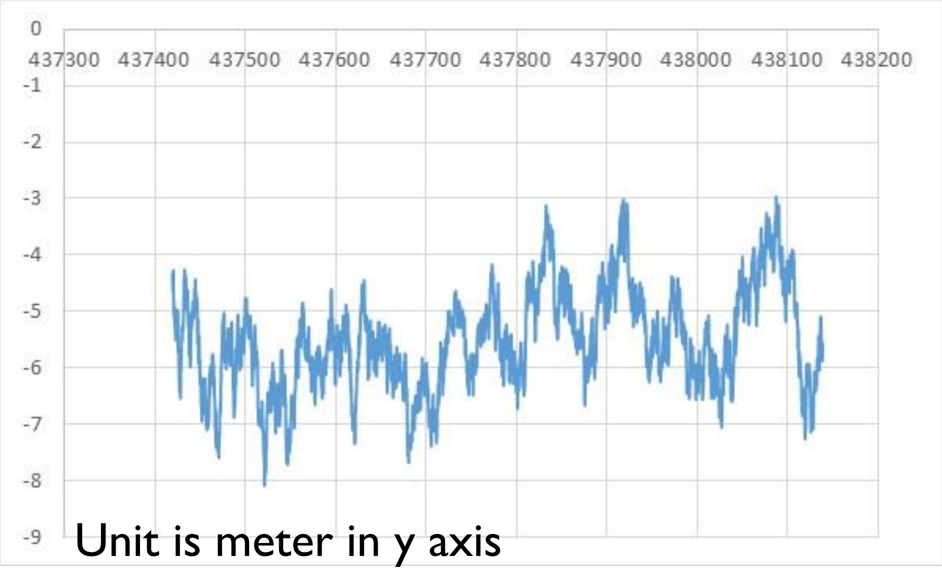
$$P_{rov\_ref}^{sv1\_sv2} = r_{rov\_ref}^{sv1\_sv2} + \varepsilon_{p,rov\_ref}^{sv1\_sv2}$$

$$\phi_{rov\_ref}^{sv1\_sv2} = r_{rov\_ref}^{sv1\_sv2} + N_{rov\_ref}^{sv1\_sv2} + \varepsilon_{\phi,rov\_ref}^{sv1\_sv2}$$

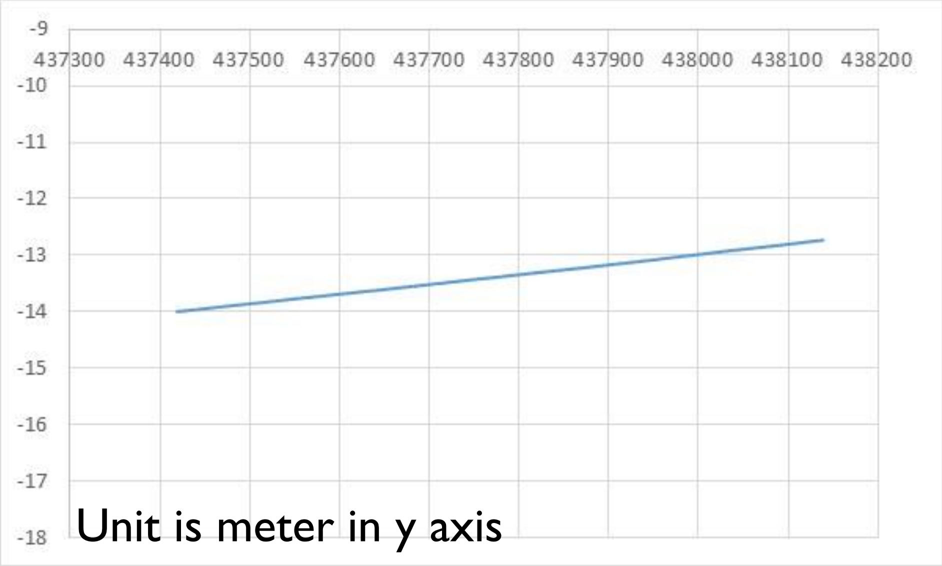
- Once you can resolve **integer N** in carrier phase double difference, you get accurate position about 1 cm.
- It can be imagine that the pseudo-range (code) accuracy is quite important.
- Code-phase is noisy (1 m-) but absolute distance
- Carrier-phase is accurate but includes integer ambiguity

3. Test results on the rooftop

- double difference of 10 m baseline-



Unit is meter in y axis



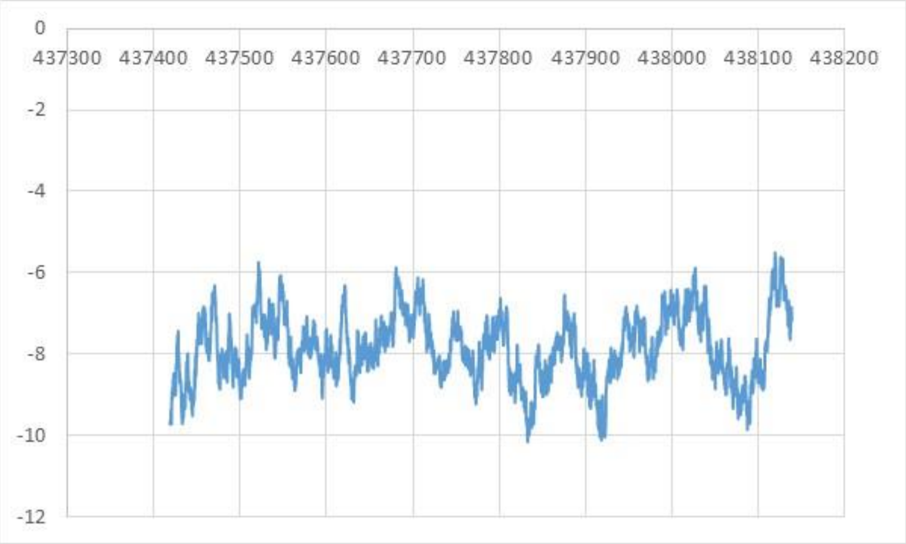
Unit is meter in y axis

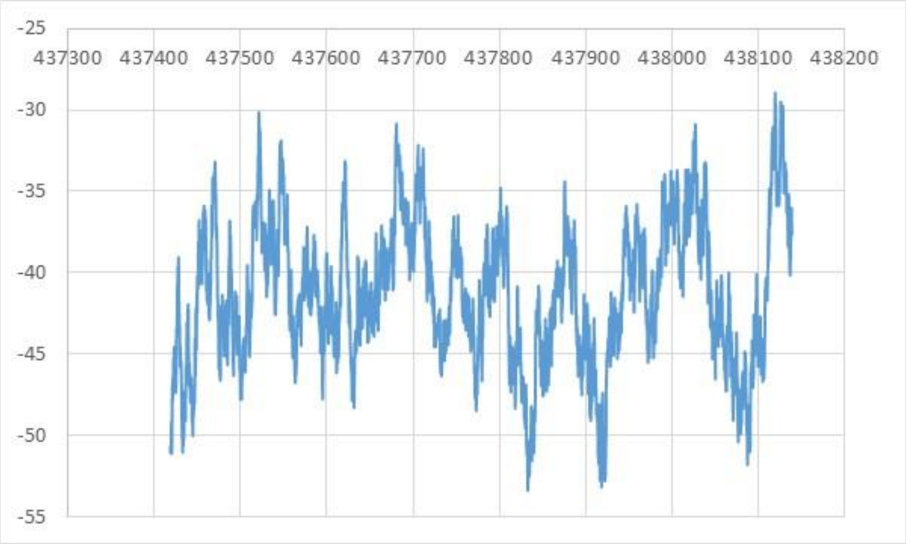
1. Reference satellite GPS PRN 16
   and target satellite is GPS PRN 8
2. Which is code-phase double difference ?
3. If you subtract from right to left, what happen ?

$$P_{rov\_ref}^{sv1\_sv2} = r_{rov\_ref}^{sv1\_sv2} + \varepsilon_{p,rov\_ref}^{sv1\_sv2}$$

$$\phi_{rov\_ref}^{sv1\_sv2} = r_{rov\_ref}^{sv1\_sv2} + N_{rov\_ref}^{sv1\_sv2} + \varepsilon_{\phi,rov\_ref}^{sv1\_sv2}$$

4. (Carrier DD) - (Code DD)





The unit is meter

Divided by wavelength
0.19029 m… (L1)

The unit is cycle

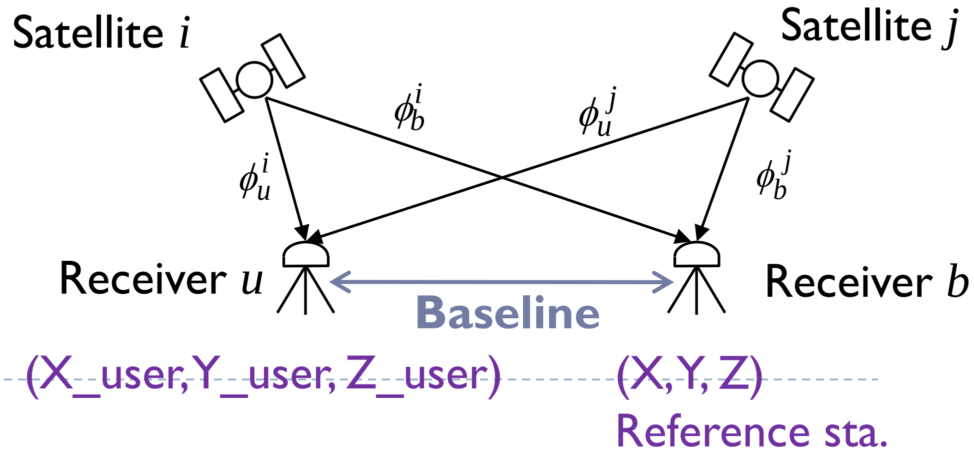
Probably, we guess the integer ambiguity between PRN16 and PRN8 is about - 40 ?
In fact, the average of this right results was - 41.3

5. What is the correct ambiguity ?

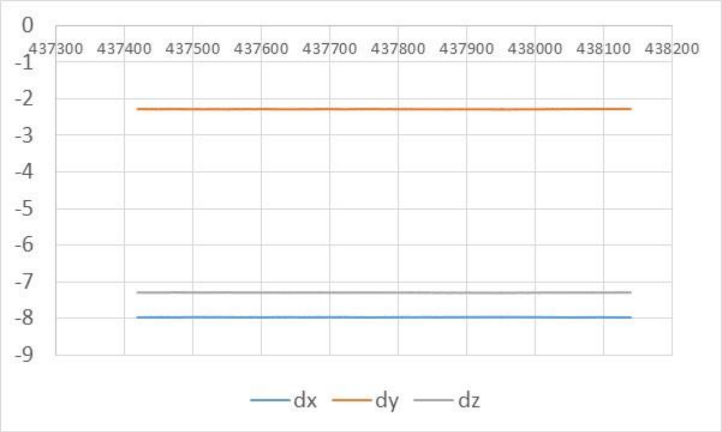
- “Integer least square method” tells us “– 42” in a single epoch !
- If you know the <u>3 or more ambiguities</u>, you can estimate the user position with the level of carrier phase because only 3 unknowns remains.
- Then, (dx, dy, dz) can be estimated and finally,
- (X_user, Y_user, Z_user) = (X, Y, Z) + (dx, dy, dz)

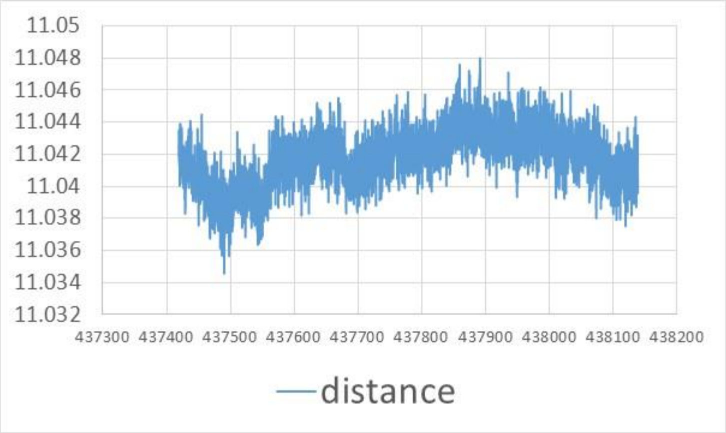
$$P^{sv1\_sv2}_{rov\_ref} = r^{sv1\_sv2}_{rov\_ref} + \varepsilon^{sv1\_sv2}_{p,rov\_ref}$$

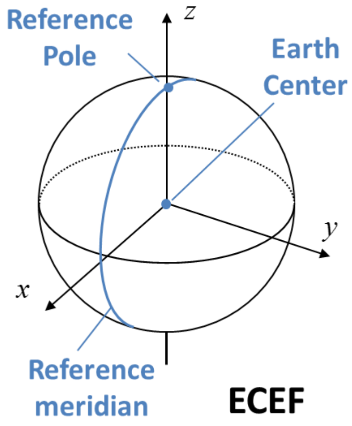
$$\phi^{sv1\_sv2}_{rov\_ref} = r^{sv1\_sv2}_{rov\_ref} + \boxed{N^{sv1\_sv2}_{rov\_ref}} + \varepsilon^{sv1\_sv2}_{\phi,rov\_ref}$$

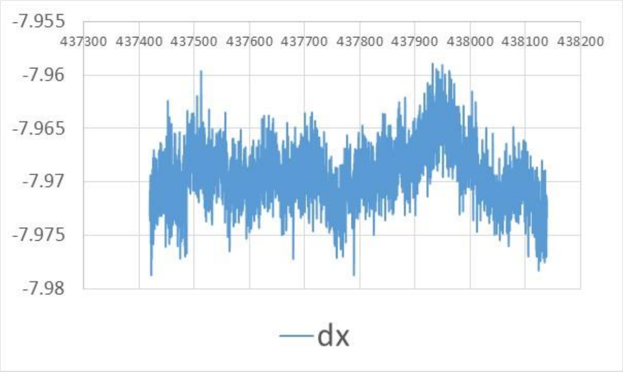


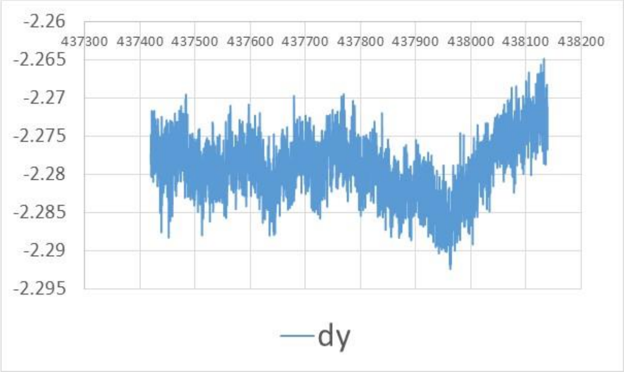
6. Test results (dx, dy, dz)

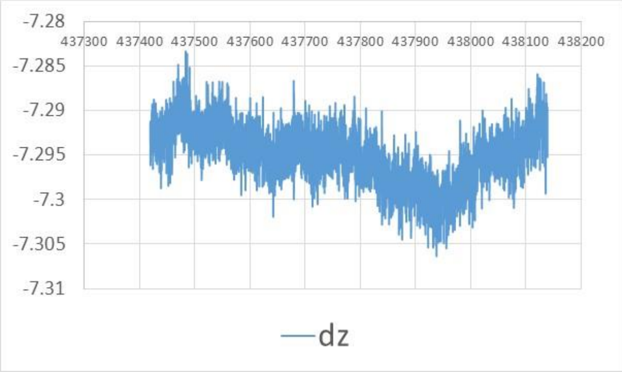










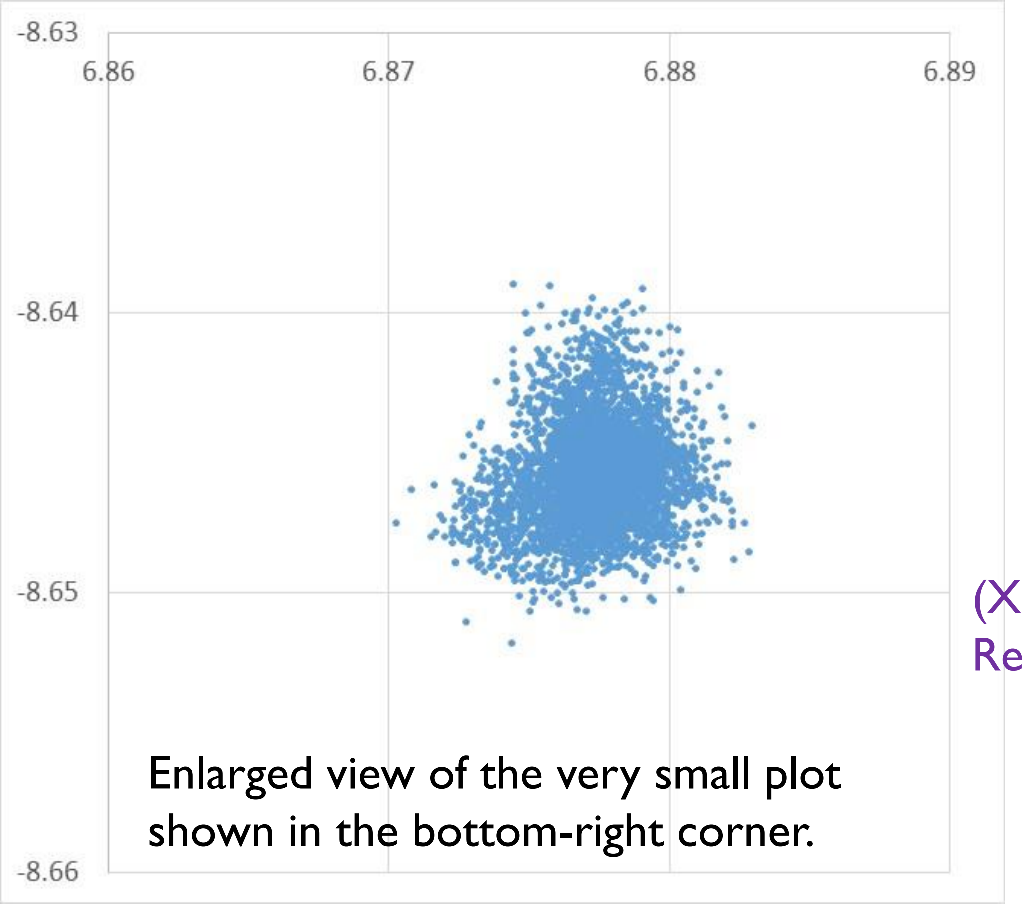


Std. = 2.8 mm

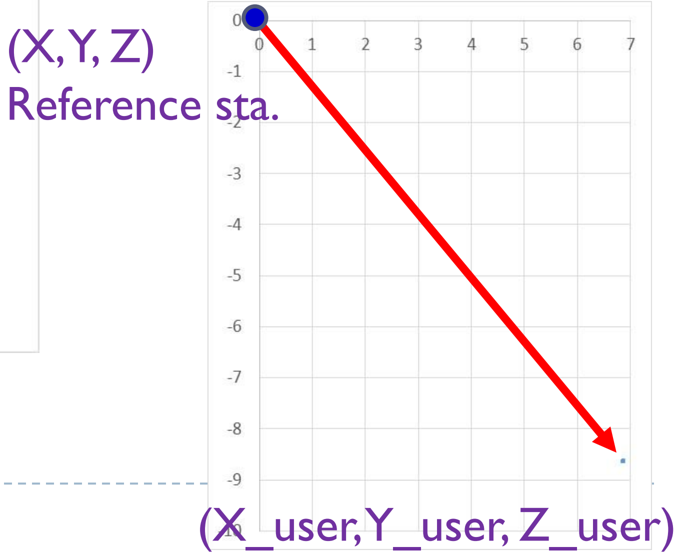
Std. = 4.0 mm

Std. = 3.4 mm

7. Convert to horizontal positions



Enlarged view of the very small plot shown in the bottom-right corner.



I am repeating myself, RTK tells you only dx, dy, dz.
You have to decide the precise reference positions !

RTKNAVI demonstration

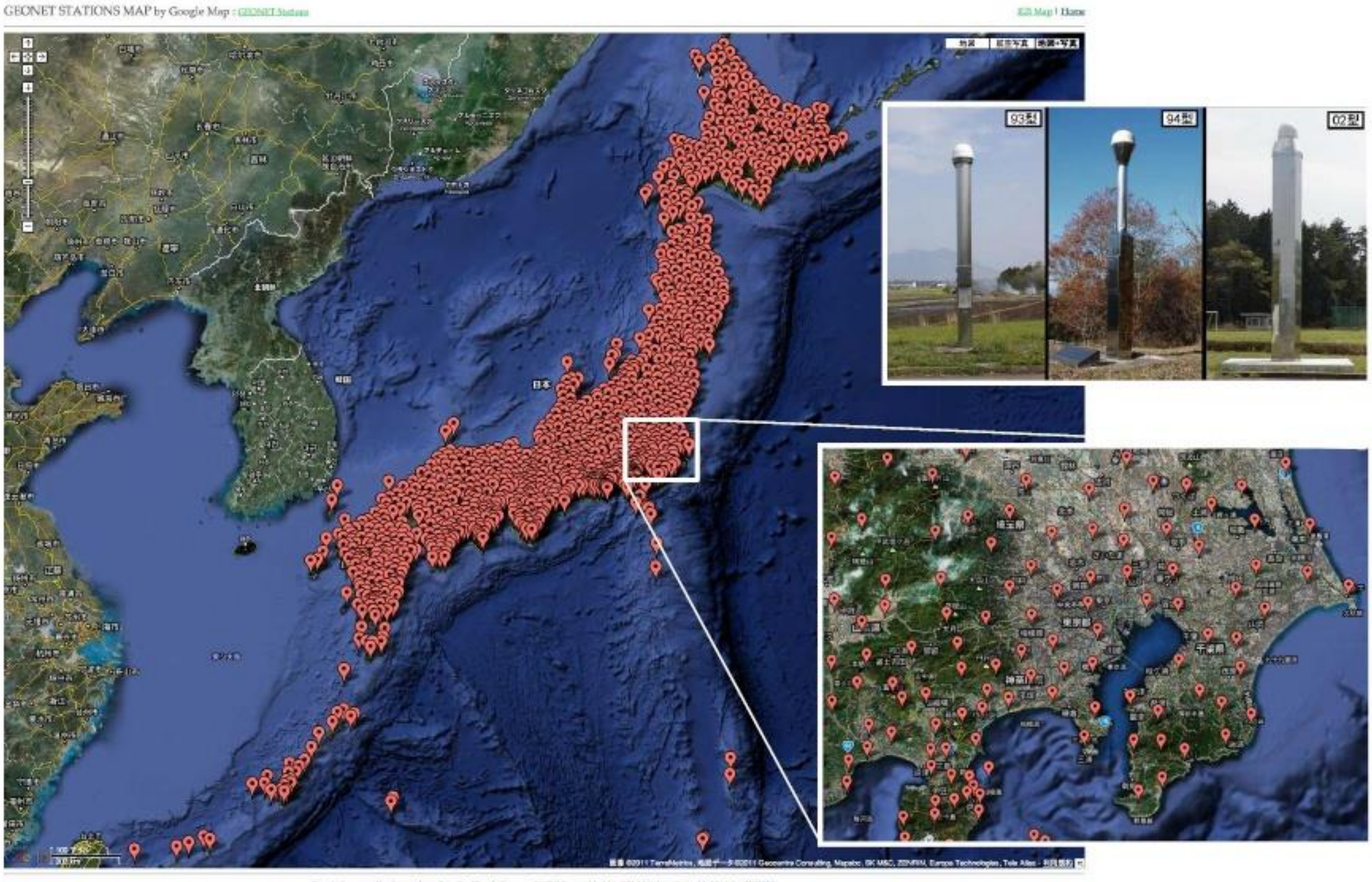
- SPP
- DGNSS
- RTK-GNSS

QZSS correction services

- See the actual performance
- GNSS TUTOR　公開

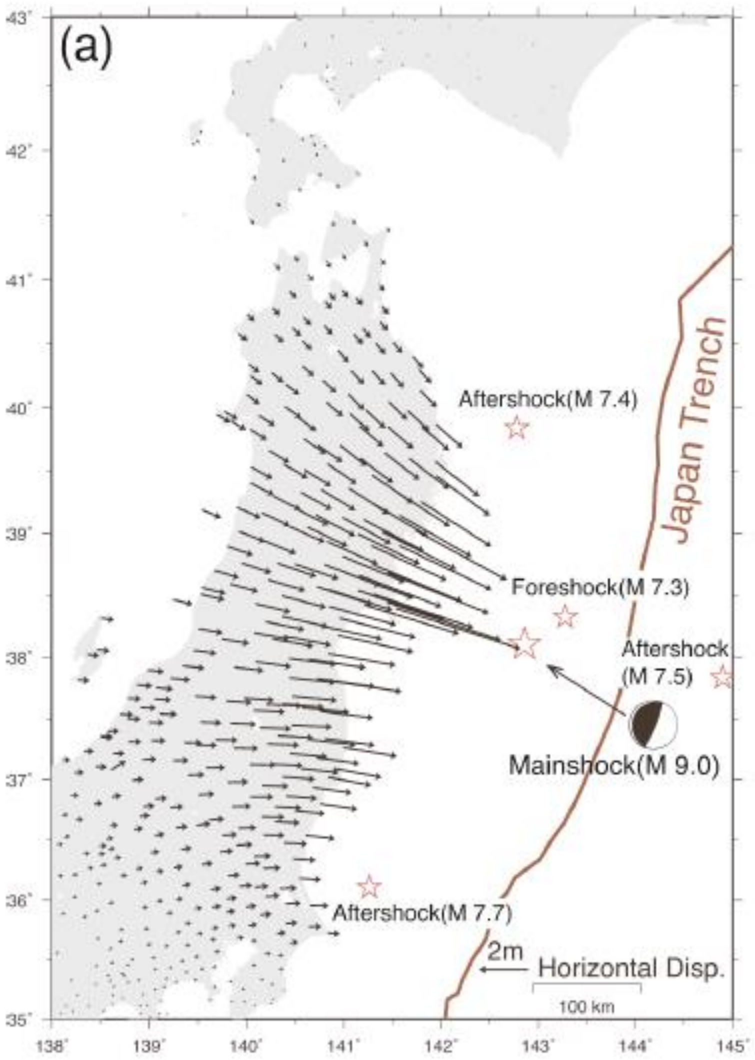
Japanese GEONET

おおむね10-30km程度以内の間隔



(http://terras.gsi.go.jp/ja/index.htm)

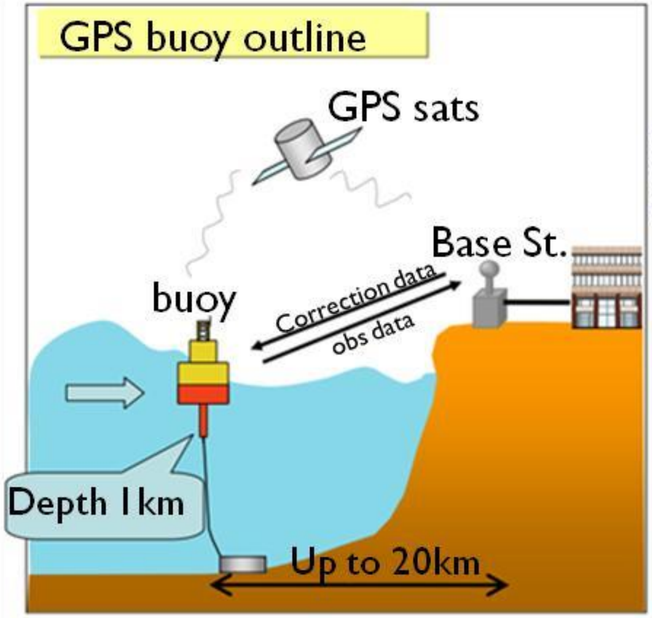
Application of RTK
(ground surface movement)



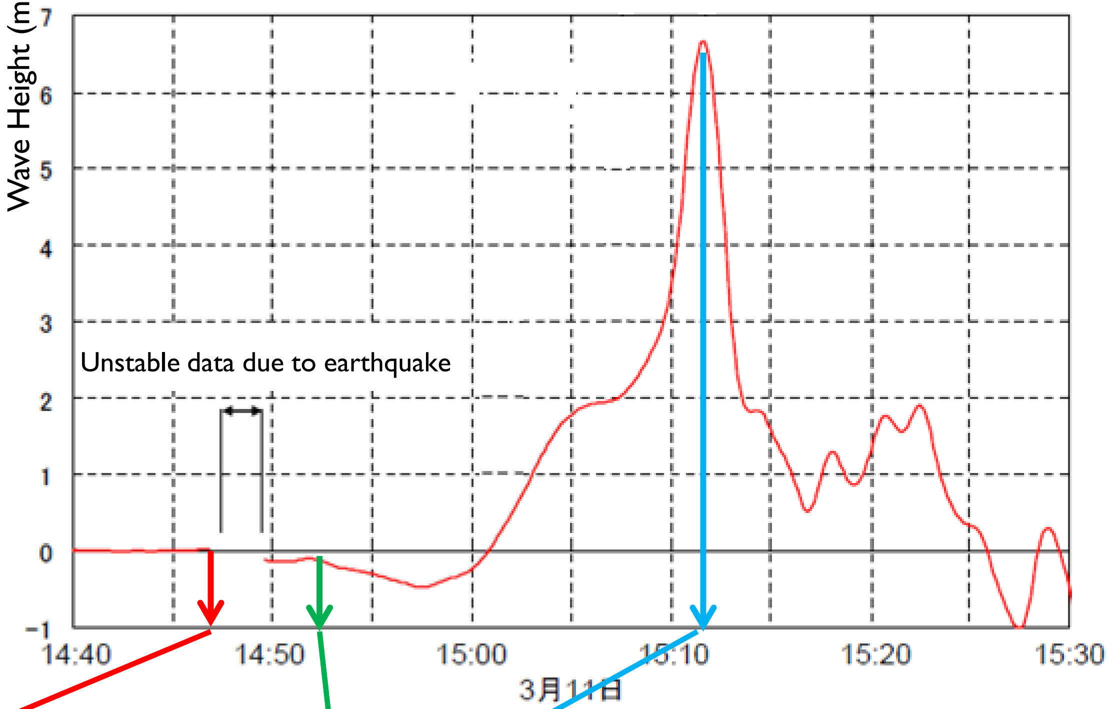
After the main shock of the March 2011 Tohoku Earthquqke, GSI tried to get initial results of ground surface movements detected by GEONET, which consists of 1,240 GNSS-based control stations and the central analysis unit. After a week, it revealed that Oshika station, close to the epicenter of the main shock, moved 5.3 m eastward and subsided 1.2 m, the largest movement ever observed by GEONET.

by Coordinates and EPS Letter

Application of RTK
(Tsunami detection)



- South *Iwate* buoy
  - 10km offshore
  - Depth 200m



- 14 : 46 Earthquake
- 14 : 53 First detected Tsunami motion
- 15 : 12 Tip of Tsunami wave

by Port and Airport Research Institute

Smart Construction

- **Computer aided construction**

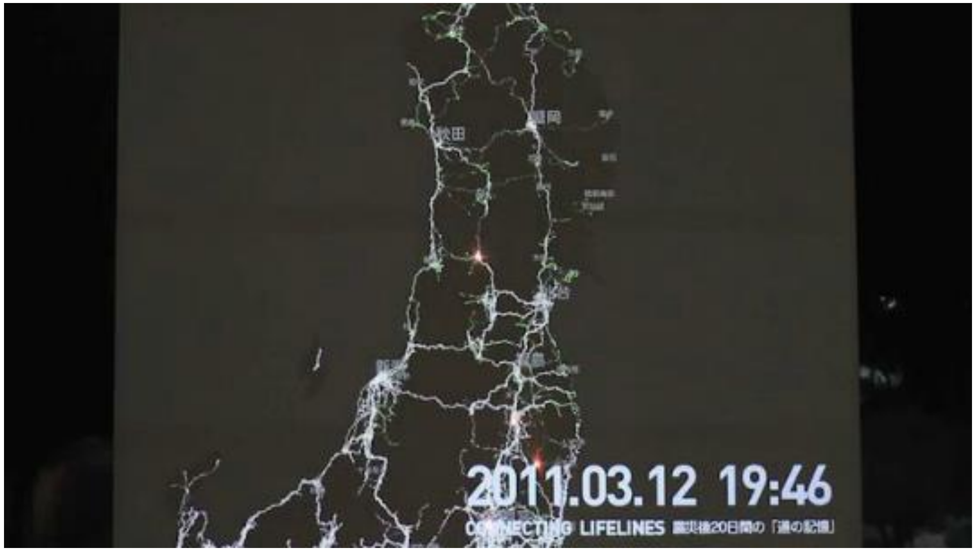
Precision Farming

- **Precision farming resolves the issue in decreasing farm family**

* Agricultural management
* Low cost receiver
* Amateur can control
* Improvement of harvest
* Improvement of quality
* Autonomous helicopter

Quality of Big data

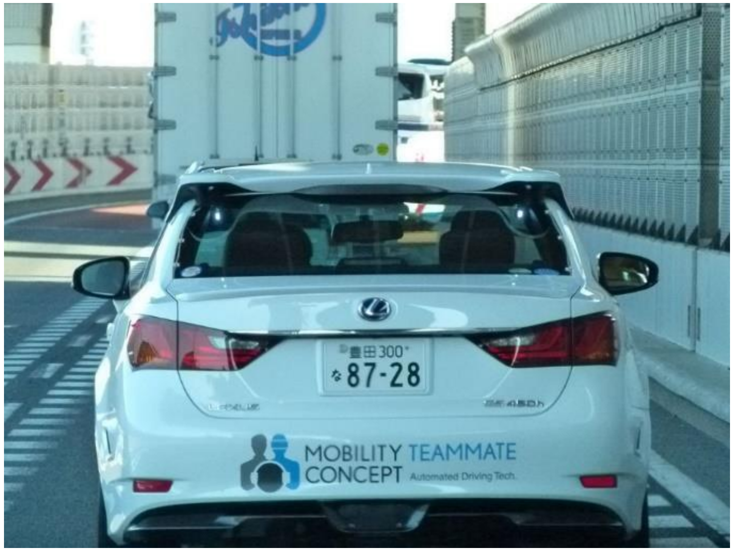
- **Road condition monitoring**
- **Traffic information in big disaster**



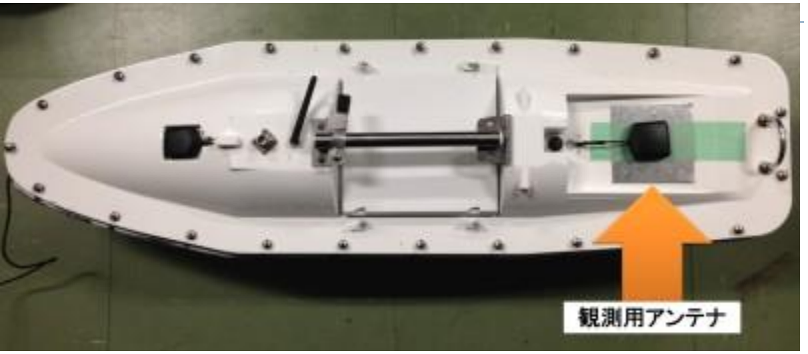
Accuracy improves the quality of Big data

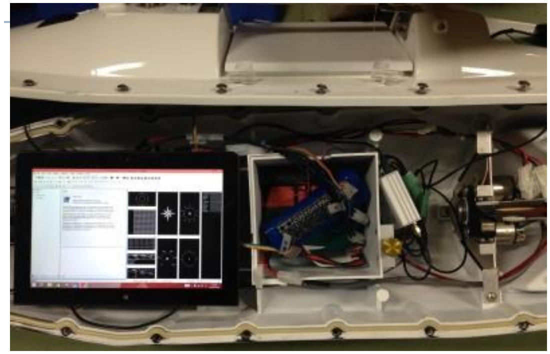
Autonomous car with precise map

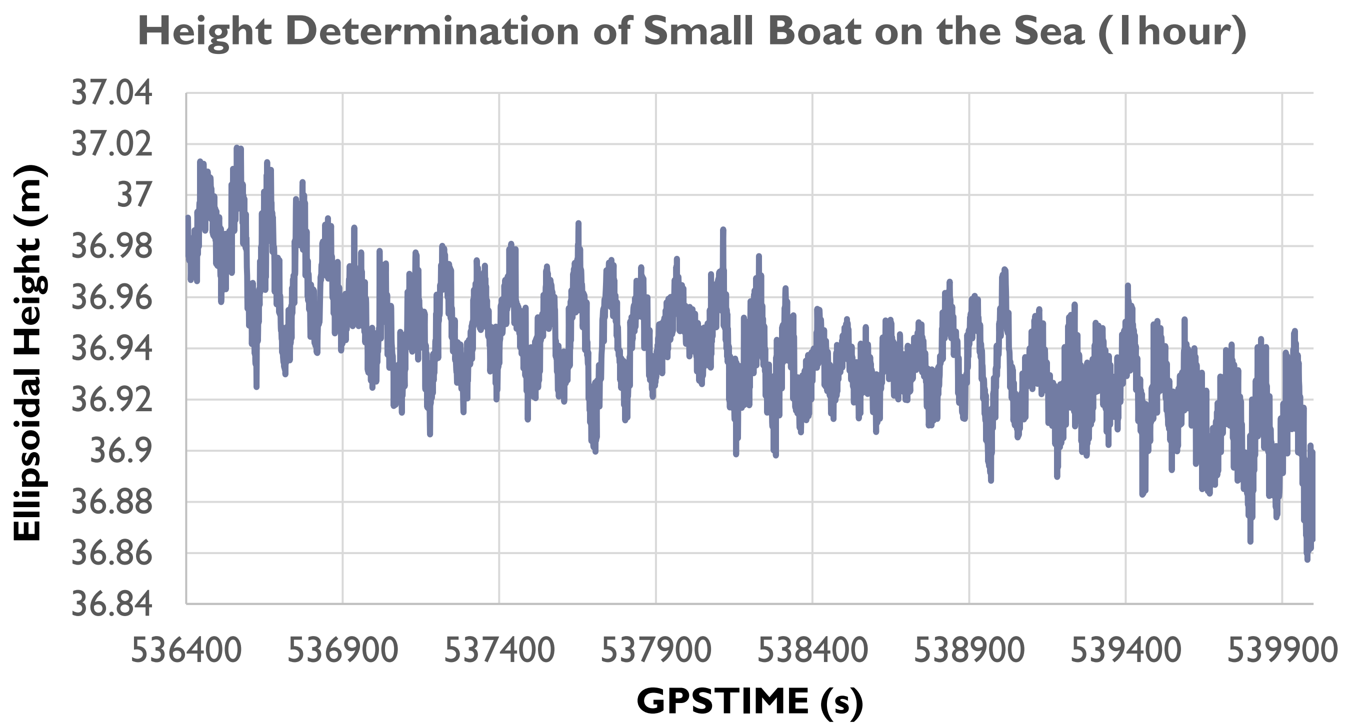
* Autonomous car
* Smart control



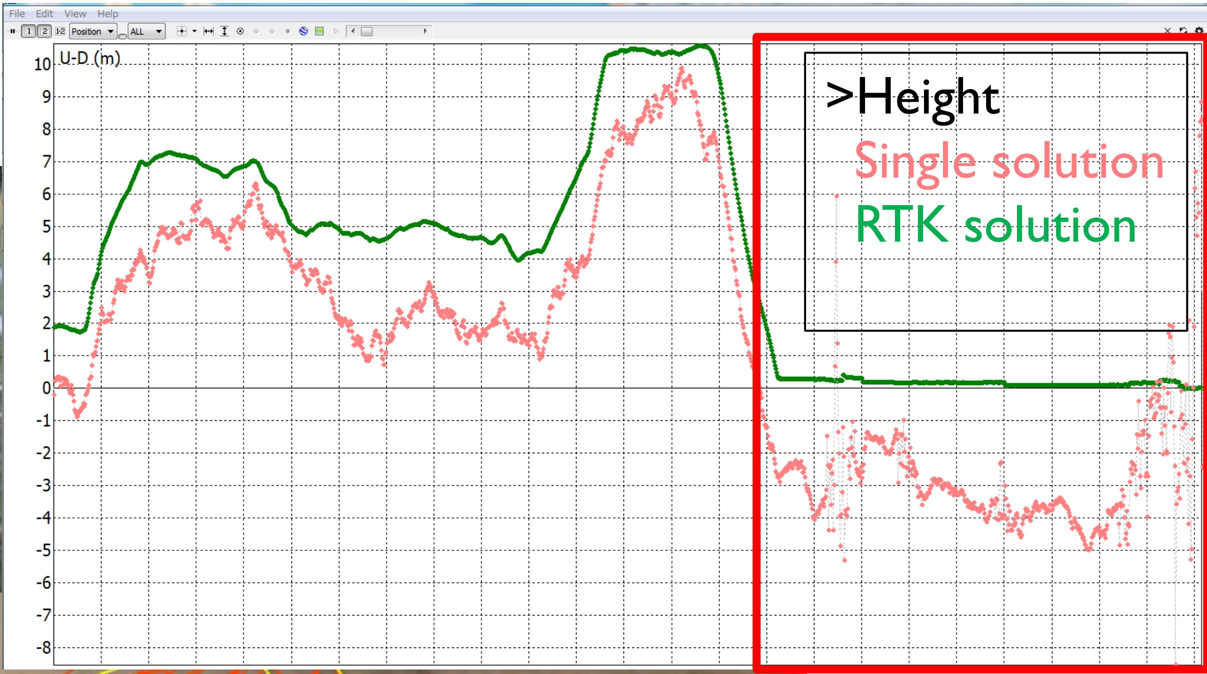
Altitude determination for boat







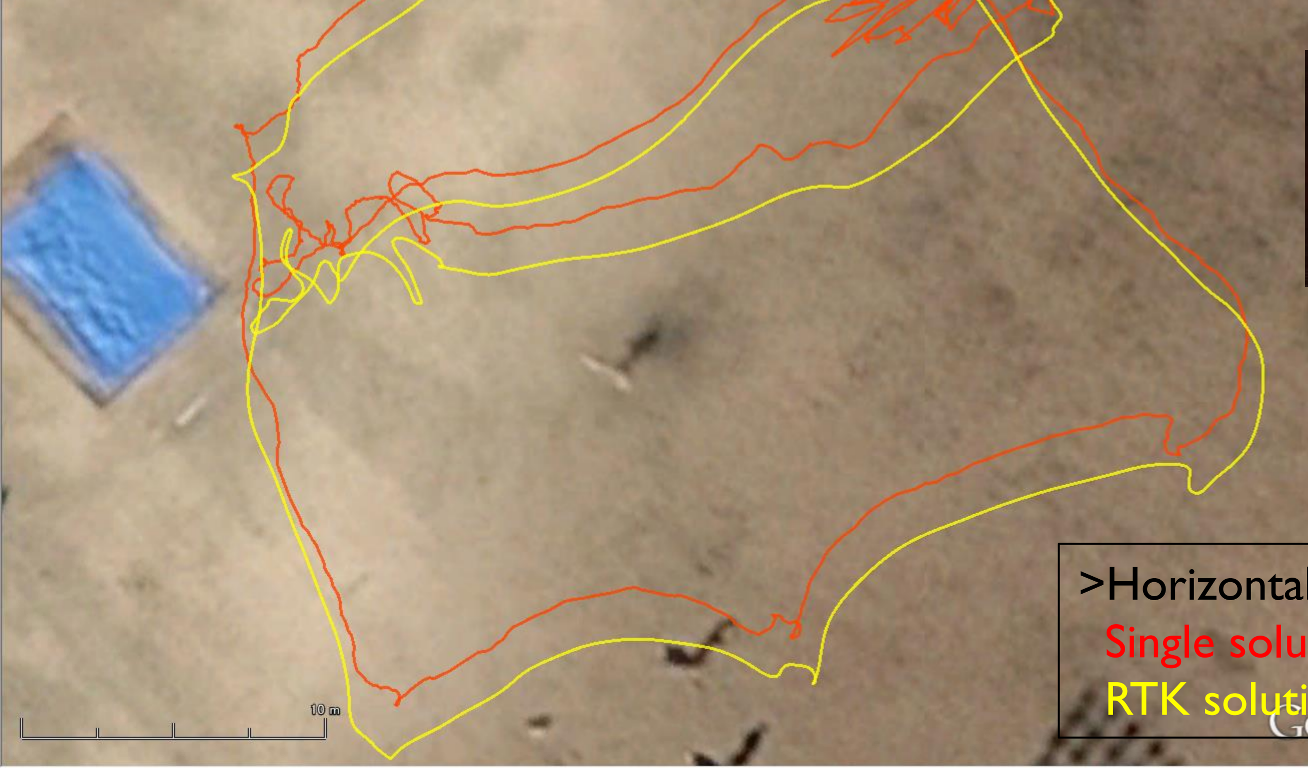
RTK for UAV



>Height

Single solution

RTK solution

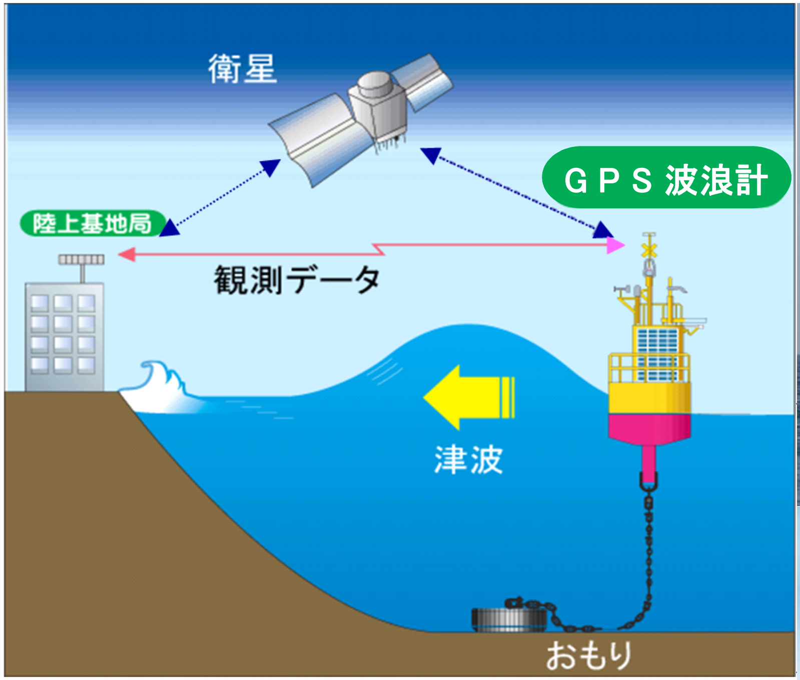


>Horizontal

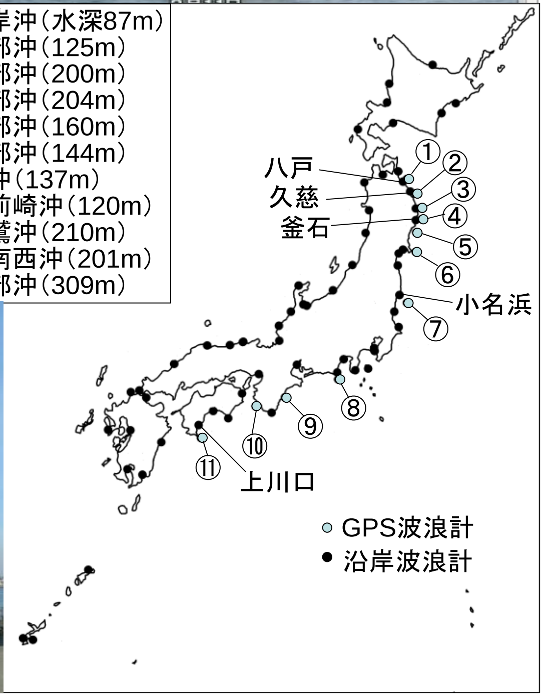
Single solution

RTK solution

Tsunami Detection



①青森東岸沖(水深87m)
②岩手北部沖(125m)
③岩手中部沖(200m)
④岩手南部沖(204m)
⑤宮城北部沖(160m)
⑥宮城中部沖(144m)
⑦福島県沖(137m)
⑧静岡御前崎沖(120m)
⑨三重尾鷲沖(210m)
⑩和歌山南西沖(201m)
⑪高知西部沖(309m)



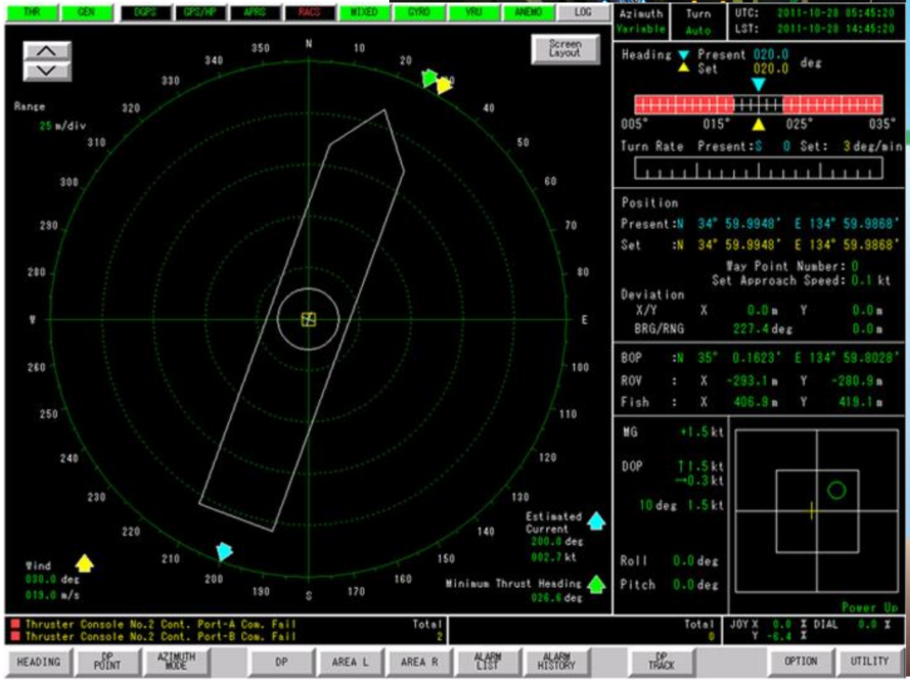
Wide Area RTK / PPP can be used

DPS (Dynamic Positioning System)

Graphic Guide
「ちきゅう」大解剖

風と波と潮で揺れる海上で
位置を保持する高度な技術 DPS
（自動船位保持システム）

2011 年 12 月掲載



Range 25 m/div

Azimuth Variable
Turn Auto
UTC: 2011-10-28 05:45:20
LST: 2011-10-28 14:45:20

Heading Present 020.0 deg
Set 020.0 deg
Turn Rate Present: S 0 Set: 3 deg/min

Position
Present : N 34° 59.9948' E 134° 59.9868'
Set : N 34° 59.9948' E 134° 59.9868'
Way Point Number: 0
Set Approach Speed: 0.1 kt

Deviation
X/Y X 0.0 m Y 0.0 m
BRG/RNG 227.4 deg 0.0 m

BOP : N 35° 0.1623' E 134° 59.8028'
ROV : X -293.1 m Y -280.9 m
Fish : X 406.9 m Y 419.1 m

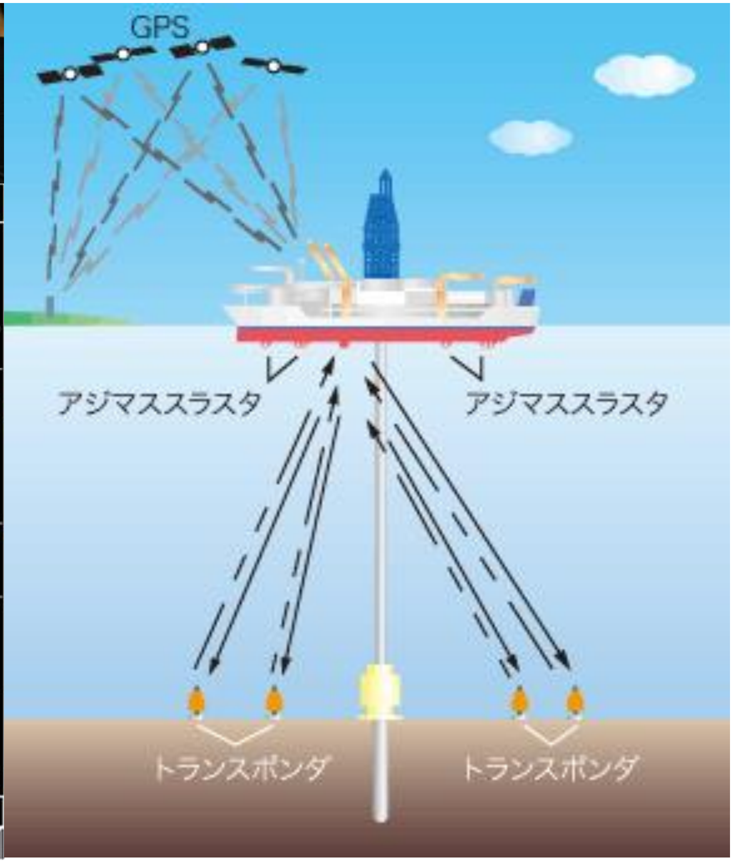
MG +1.5 kt
DOP ↑1.5 kt →0.3 kt
10 deg 1.5 kt
Roll 0.0 deg
Pitch 0.0 deg

Estimated Current 200.0 deg 002.7 kt
Wind 030.0 deg 019.0 m/s
Minimum Thrust Heading 026.6 deg

Thruster Console No.2 Cont. Port-A Com. Fail
Thruster Console No.2 Cont. Port-B Com. Fail
Total 2

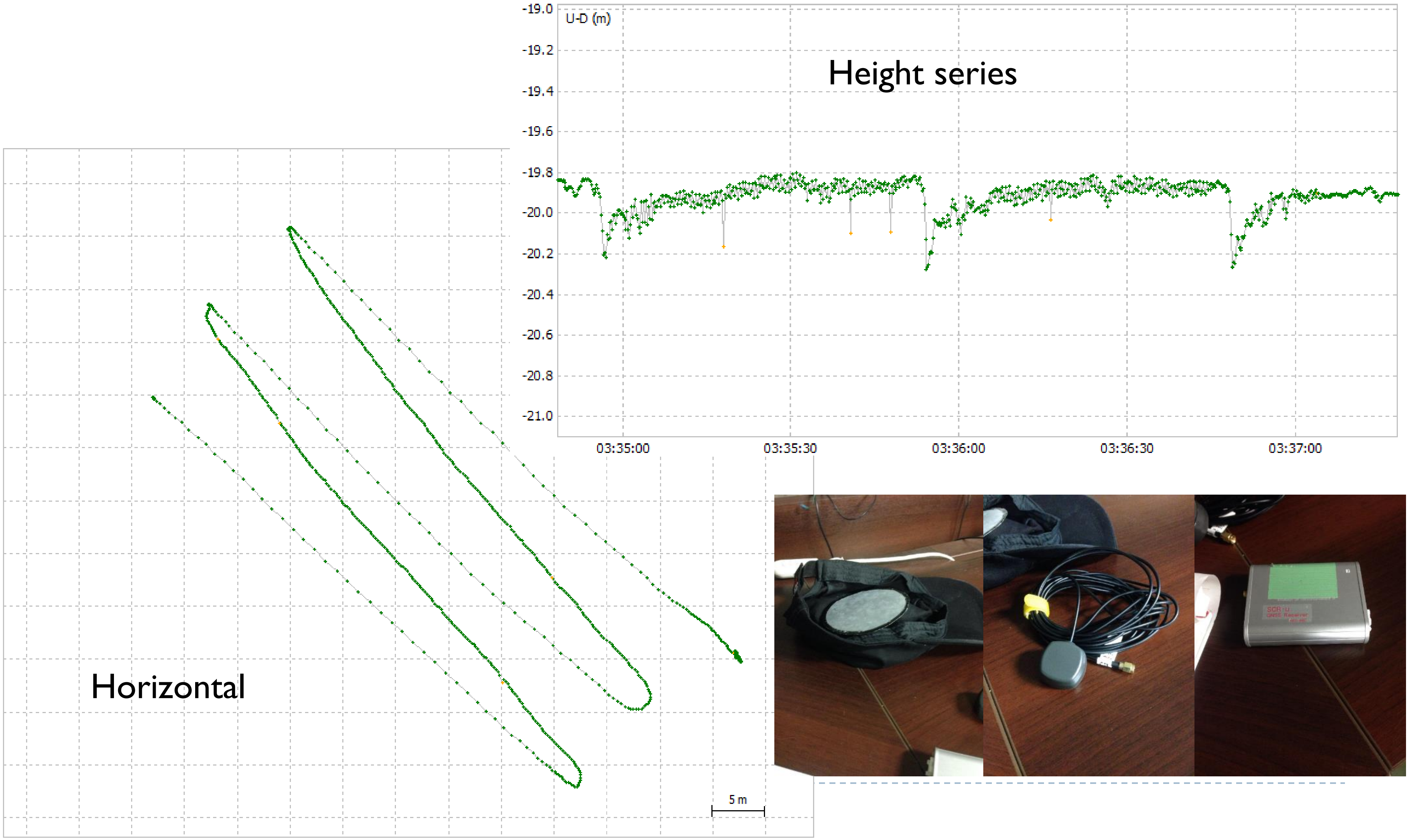
Total 0
JOY X 0.0 % DIAL 0.0 %
Y -6.4 %

Power Up



*Wide Area RTK / PPP / DFMC SBAS can be used
*50 cm (2drms) is required.
*DPS is the key technology for autonomous Ship

Recent Test : Running



Height series

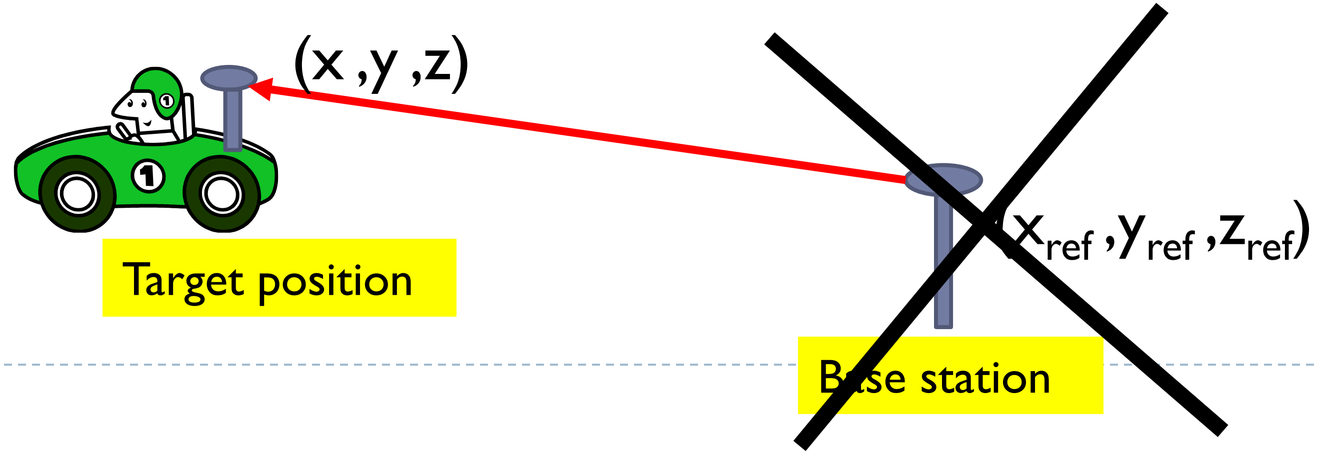
Horizontal

Difference between expensive and low-cost receiver

	Survey-grade receiver	**Low-cost receiver**
Cost	$ 100,00〜	$100〜
Multiple GNSS	Perfect	BeiDou or Glonass Other are OK
Multiple Frequency	Perfect	L1/B1/E1/G1 only
Number of channel	400-500-	-100
RTK (short baseline) + open sky	**Perfect**	**Almost perfect**
RTK (over 20 km baseline) + open sky	**Almost perfect up to 100 km or more**	**Impossible**
RTK under mid obstructed area (short)	**Almost perfect**	**May be difficult**
RTK under dense obstructed area (short)	**Sometimes not good**	**Difficult**
Accuracy of fixed position + open	mm	→
Accuracy of code position + open	Deci-meter	1-2 meter

PPP (Precise Point Positioning)

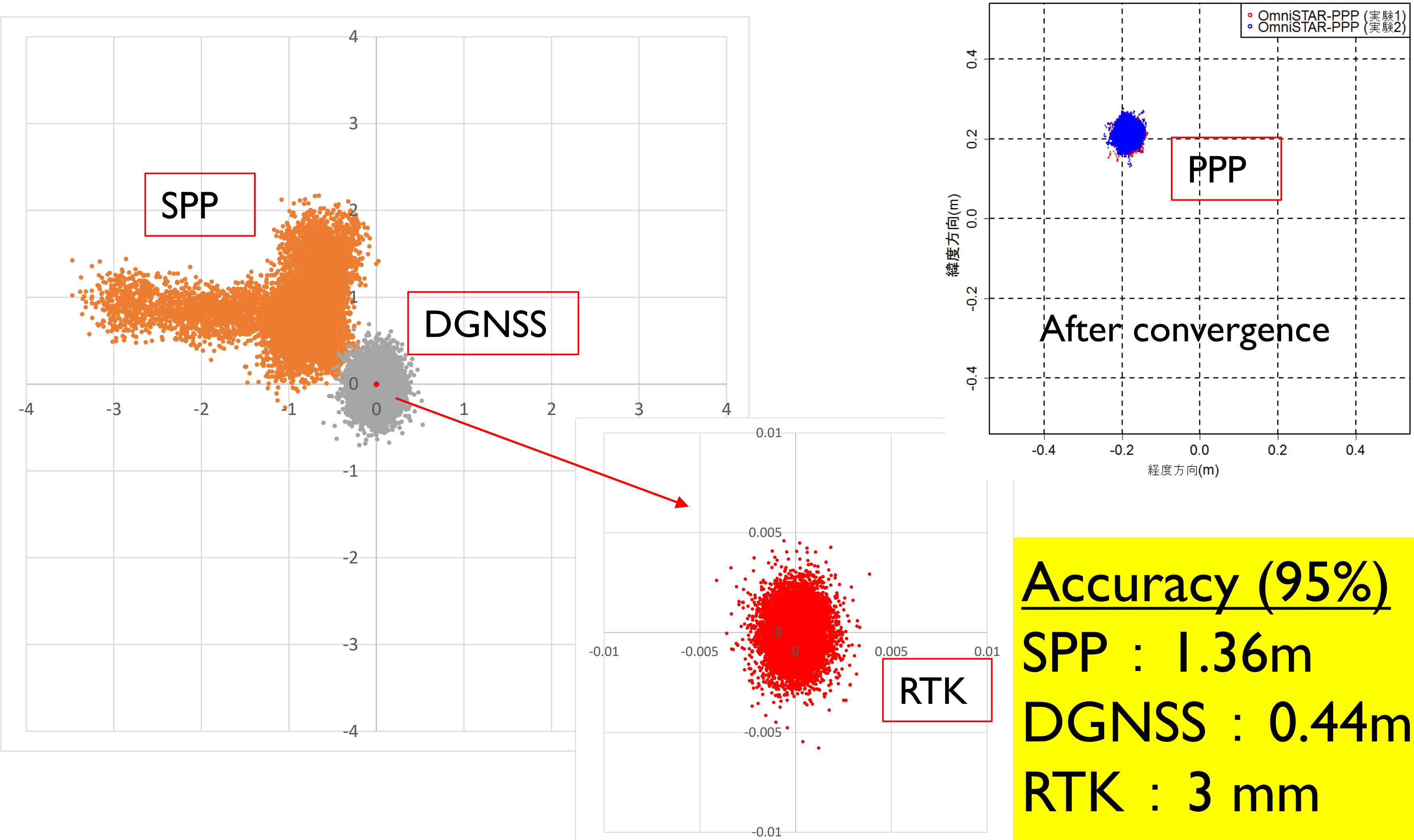
- We discussed about relative positioning to cancel the common errors.
- We switch from relative positioning to point positioning. Base station is not required.
- We need to consider the measurement errors more in details if we want to have centimeter-level accuracy in the point positioning.



PPP mitigates…

Source	Potential error size	Error mitigation
Satellite clock model	2 m (rms)	Centimeter level precise satellite clock
Satellite ephemeris prediction	2 m (rms) along the LOS	Centimeter level precise satellite position
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	Dual-frequency can mitigate it completely
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	Precise model with centimeter level
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	Carrier-phase is used
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	Carrier-phase is used

Actual performance…



Accuracy (95%)

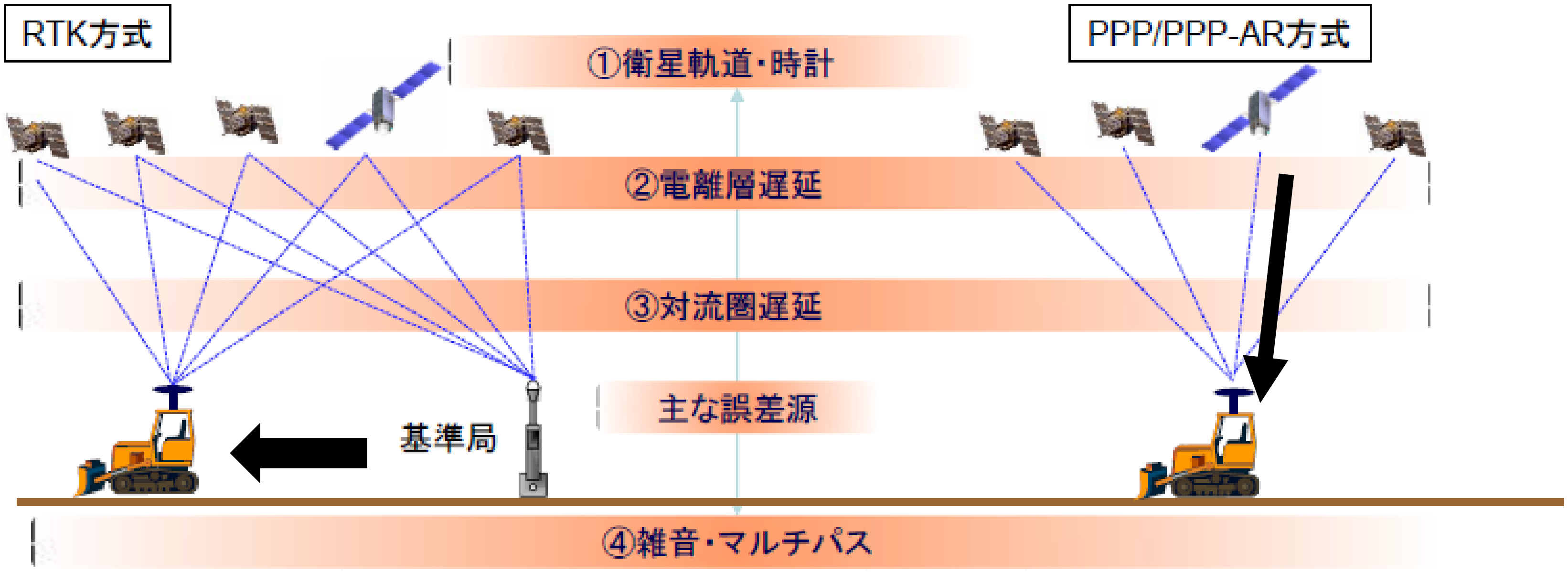
SPP ： 1.36m

DGNSS ： 0.44m

RTK ： 3 mm

PPP ： 3.4 cm

RTK and PPP



Reference station

Continuous communication

Instantaneous position

10-100km limitation

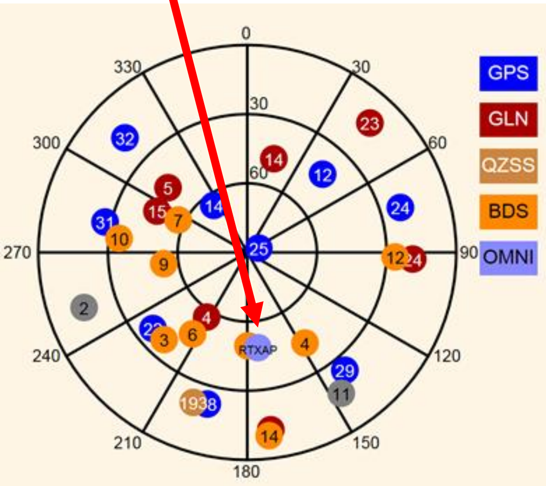
Continuous communication through satellite

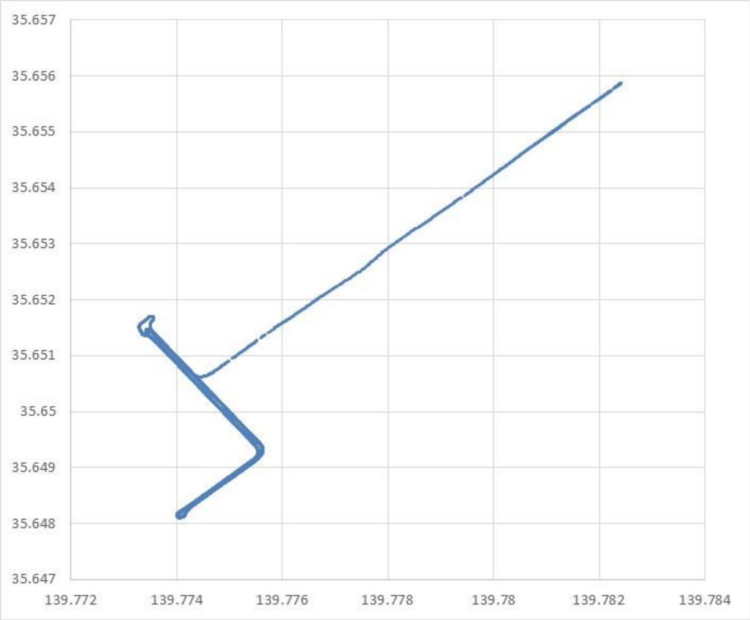
Wait for 5-30 minutes

No limitation

Precise Point Positioning Test using commercial service

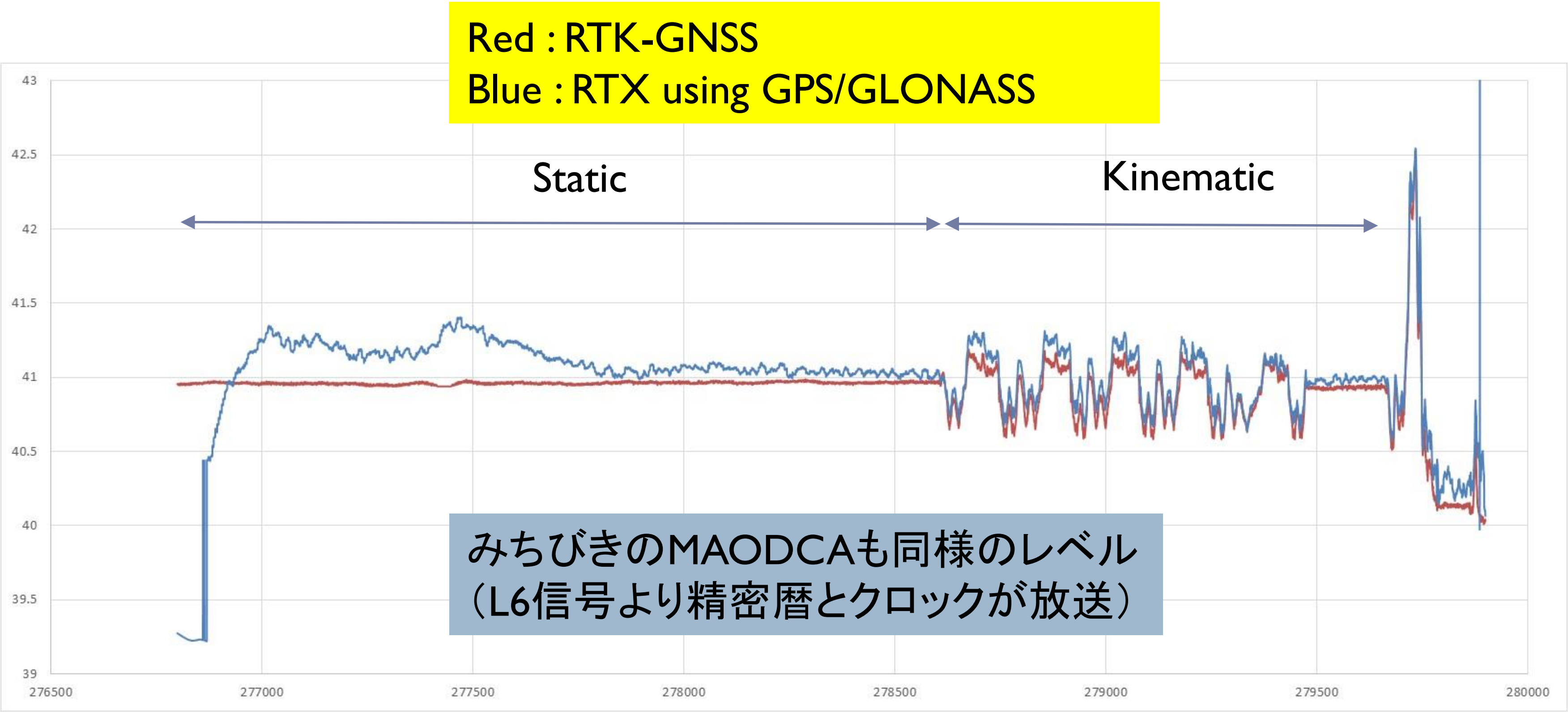
- 30 minutes static and 15 minutes kinematic
- Trimble SPS855+RTX (PPP) option
- Comparison with RTK results
- Omni-star was used
- Open Sky





Horizontal plots at Harumi Area

Altitude Comparison between RTK and RTX (PPP)



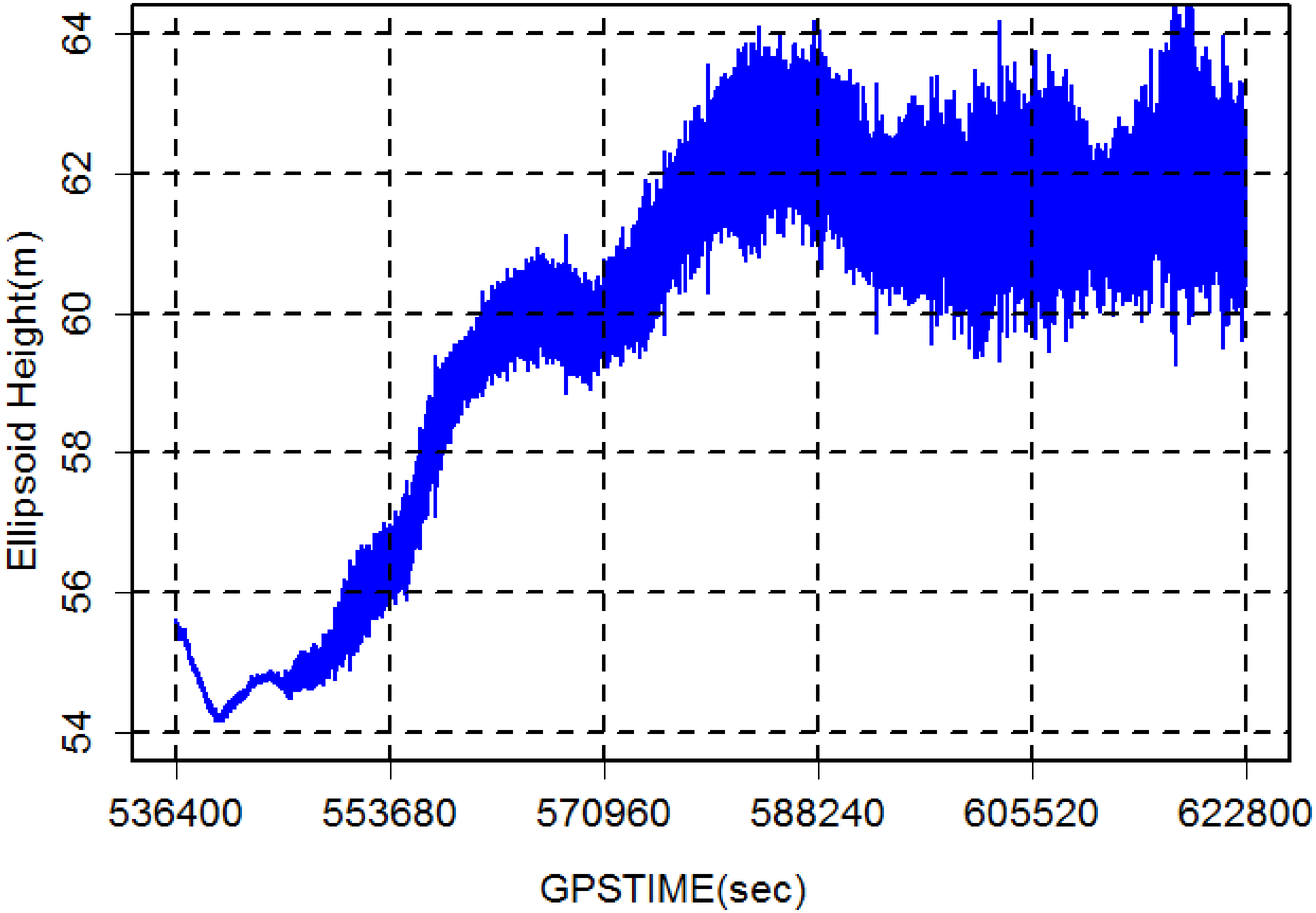
Red : RTK-GNSS
Blue : RTX using GPS/GLONASS

みちびきのMAODCAも同様のレベル
（L6信号より精密暦とクロックが放送）

The accuracy was maintained within several centi-meters after 15 minutes of power on. Small bias (about 10cm) was deduced from other reason.

PPP Positioning Results of Height for SHIP

- 2015.7.25. (Tokyo ~ Ogasawara)



What is RTCM ?

- The standard for differential global navigation satellite system was defined in RTCM Special Committee 104 and its current version is Version 3. RTCM standard for differential global navigation satellite services are communication protocols between reference stations and mobile receivers which allow very high accurate positioning, when compared with positioning system without augmentation.

What is NTRIP ?

- The NTRIP was also defined in the RTCM Special Committee 104. NTRIP stands for "Networked Transport for RTCM via Internet Protocol". It is based on Hypertext transfer Protocol version 1.1 and the intention is to disseminate differential correction data through the internet.

https://igs.bkg.bund.de/index/index